Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 1

Name:_____

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts)
Let $\mathbf{A}$ be the $n \times n$ matrix of costs for the asignment problem. Suppose that the entries $a_{i,j}$ are independent exponential mean 1 random variables. Let C be the indices of the columns of $\mathbf{A}$. The GREEDY algorithm proceeds as follows to produce a perfect matching M of cost Z:

GREEDY

begin
$S \leftarrow C$; $M \leftarrow \emptyset$
for $i = 1, 2, \ldots, n$ **do**
begin
Let $Z_i = a_{i,k} = \min\{a_{i,j}, j \in S\}$, choose the cheapest available edge for i.
$S \leftarrow S \setminus \{k\}$; $M \leftarrow M \cup \{(i,k\}$, remove k from the available choices.
end;
Output: M and $Z = Z_1 + \cdots + Z_n$.
end

1. How are the Z_i distributed?
2. What is $\mathbb{E}Z_i$?
3. What is $\mathbb{E}Z$?

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Q2: (33pts)
$\mathcal{X} = \{X_1, X_2, \ldots, X_n\}$ are n points randomly chosen from the unit cube $[0,1]^3$ in $\mathbb{R}^3$. Let Z_{TSP} denote the minimum cost of a traveling salesperson tour through $\mathcal{X}$. Show that w.h.p.,

$$Z_{TSP} \geq c_1 n^{2/3},$$

for some absolute constant $c_1 > 0$.

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Q3: (34pts)
$\mathcal{X} = \{X_1, X_2, \ldots, X_n\}$ are n points chosen from the unit cube $[0,1]^3$ in $\mathbb{R}^3$. Let Z_{TSP} denote the minimum cost of a traveling salesperson tour through $\mathcal{X}$. Show that

$$Z_{TSP} \leq c_2 n^{2/3},$$

for some absolute constant $c_2 > 0$.

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