Department of Mathematical Sciences
Carnegie Mellon University

21-393 Operations Research II
Test 1

Name:_____

Problem	Points	Score
1	35	
2	35	
3	30	
Total	100	

Q1: (35pts)
(a) Fill in the last column of the table below for solving the following knapsack problem and produce an optimal solution:

$$\begin{array}{lrcrcrcl} \text{maximise} & 3x_1 & + & 7x_2 & + & 16x_3 & & \\ \text{subject to} & 2x_1 & + & 3x_2 & + & 6x_3 & \leq & 12 \end{array}$$

$x_1, x_2, x_3 \geq 0$ and integer.

w	$f_1(x_1)$	b_1	$f_2(x_2)$	b_2	$f_3(x_3)$	b_3
0	0	0	0	0		
1	0	0	0	0		
2	3	1	3	0		
3	3	1	7	1		
4	6	1	7	1		
5	6	1	10	1		
6	9	1	14	1		
7	9	1	14	1		
8	12	1	17	1		
9	12	1	21	1		
10	15	1	21	1		
11	15	1	24	1		
12	18	1	28	1		

(b) Read off the solution to the problem below from the above table.

$$\begin{array}{lrcrcrcl} \text{minimise} & 2x_1 & + & 3x_2 & + & 6x_3 & & \\ \text{subject to} & 3x_1 & + & 7x_2 & + & 16x_3 & \geq & 20 \end{array}$$

$x_1, x_2, x_3 \geq 0$ and integer.

Q2: (35pts)

A factory produces three products X,Y,Z. Only products X,Y are sold and there are monthly demands of d_X, d_Y for these products, respectively. It requires one unit of product Z, at least one period old, to make each unit of product Y. The factory can only make one type of item in a period. It costs $c_X(x), c_Y(x), c_Z(x)$ respectively to make x units of X,Y,Z respectively. Assume there is a maximum storage of H allowed and initially there are $H/3$ items of each in store. Write down a dynamic programming recurrence to solve the problem. There is also a storage charge of I per unit stored in any period.

Assume also that H is large enough so that there are no questions of feasibility.

Q3: (30pts)
Formulate the following as an integer program: n objects are to be placed at the points $1, 2, \ldots, n$ on a circle. For each object i there is a set S_i of objects that cannot be placed adjacent to i. I.e. if object i is placed at position t and $j \in S_i$ then object j cannot be placed at $t-1$ or $t+1$. (position $n+1$ is position 1 here.) Write down a set of inequalities in integer variables that can be used to determine a way of placing the objects.