Hospital Construction Planning

An Optimization-Based Approach to Hospital Capacity Design

Jocelyn Wang
Julia Jiao
Haibo Zhu
Isabel Khan

Department of Mathematical Sciences
Carnegie Mellon University

Operations Research II
Professor: A.M.Frieze

December 15, 2025

1 Introduction & Scope

1.1 Background

Planning a new hospital facility involves long-term infrastructure decisions that are difficult and costly to modify once construction is complete. Hospitals must contend with substantial upfront capital expenditures, uncertainty in future patient demand, constrained operating budgets, and strict regulatory and safety requirements. These factors make ad-hoc or purely experience-based planning approaches risky. An inefficient allocation of room types may result in underutilized resources, unmet patient needs, or sustained financial inefficiencies over the facility's lifetime.

1.2 Model Goal and Description

To address these challenges, we develop a linear programming model that provides a unified optimization framework for hospital room planning. The objective of the model is to determine how many rooms of each type a new hospital facility should construct in order to maximize long-run annual net profit, subject to financial, operational, and demand constraints.

The model explicitly balances revenue generation and cost control by incorporating construction costs, operating costs, and per-patient revenues, while automatically enforcing budgetary, capacity, and demand limitations. By doing so, it generates an optimal mix of room types and offers a structured and transparent decision-support tool for evaluating trade-offs in hospital facility planning.

2 Data

2.1 Cost of building each type of room

Room Type	Build (M$)	Machine (M$)	Annual Maint. (M$)	Capacity (patients/yr)
Operating Room (OR)	4.0	1.2	0.50	700
ICU Room (ICU)	2.5	0.4	0.30	500
General Ward (WARD)	0.8	0.1	0.10	1200
Imaging Room (IMAGING)	3.0	2.0	0.40	5000
Clinic Room (CLINIC)	0.6	0.2	0.08	4000

2.2 Profit per patient (revenue − per-patient variable cost)

Room Type	Profit/Patient (M$)
Operating Room (OR)	0.0050
ICU Room (ICU)	0.0020
General Ward (WARD)	0.0010
Imaging Room (IMAGING)	0.0005
Clinic Room (CLINIC)	0.0003

2.3 Annual demand for each type of room

Room Type	Annual Demand (Number of Patients)
Operating Room (OR)	7000
ICU Room (ICU)	4000
General Ward (WARD)	15000
Imaging Room (IMAGING)	30000
Clinic Room (CLINIC)	20000

2.4 Key parameters suppositions

- Construction and equipment budget $B = 150$ M$,
- Annual maintenance and electricity budget $M = 25$ M$,

- Hospital planning across a horizon of $H = 10$ years.

3 LP Parameters, Constraints, and Objective

We formulate the hospital construction planning problem as a linear program (LP) that determines the optimal number of rooms of each type to construct and the corresponding annual patient throughput. The model incorporates capital investment, operating costs, capacity limitations, and patient demand, and seeks to maximize long-term annual net profit.

3.1 Sets and Decision Variables

Let

$$J = \{\text{OR}, \text{ICU}, \text{WARD}, \text{IMAGING}, \text{CLINIC}\}$$

denote the set of room types.

For each room type $j \in J$, we define the following decision variables:

- x_j: number of rooms of type j to construct,
- y_j: number of patients served per year using room type j.

The variables x_j represent long-term infrastructure decisions and are required to be nonnegative integers, while y_j are continuous nonnegative variables, representing annual patient volume.

3.2 Parameters

For each type $j \in J$, we consider the following parameters:

- build_j: construction cost per room (M$),
- machine_j: equipment cost per room (M$),
- maint_j: annual maintenance and electricity cost per room (M$),
- capacity_j: maximum number of patients served per room per year,
- demand_j: annual patient demand for room type j,
- p_j: net profit per patient (M$).

In addition, the model includes the following global parameters:

- B: total construction and equipment budget (M$),
- M: annual maintenance and electricity budget (M$),
- H: planning horizon (years), used to annualize fixed construction costs.

3.3 Objective Function

The objective of the model is to maximize *annual net profit*, defined as total variable profit from patients served subtracted by annual operating costs and annualized fixed construction costs.

Variable Profit Serving y_j patients per year in room type j generates $p_j y_j$ million dollars in annual profit. Summing across all room types, total variable profit is

$$\sum_{j \in J} p_j y_j.$$

Annual Maintenance Cost Each room of type j requires an annual maintenance and electricity cost of maint_j. If x_j rooms are constructed, the total annual maintenance cost is

$$\sum_{j \in J} \text{maint}_j x_j.$$

Annualized Fixed Construction Cost The one-time construction and equipment cost for room type j is $\text{build}_j + \text{machine}_j$. To incorporate this fixed investment into the annual objective, the total cost is spread evenly over a planning horizon of H years:

$$\frac{1}{H} \sum_{j \in J} (\text{build}_j + \text{machine}_j) x_j.$$

Final Objective Combining all components, the LP objective function is

$$\max Z = \sum_{j \in J} p_j y_j - \sum_{j \in J} \text{maint}_j x_j - \frac{1}{H} \sum_{j \in J} (\text{build}_j + \text{machine}_j) x_j.$$

This formulation balances short-run profitability against long-run capital and operating costs.

3.4 Constraints

Construction and Equipment Budget Constraint Total spending on construction and equipment cannot exceed the available capital budget:

$$\sum_{j \in J} (\text{build}_j + \text{machine}_j) x_j \leq B.$$

Annual Maintenance Budget Constraint The total annual maintenance and electricity cost across all rooms must not exceed the operating budget:

$$\sum_{j \in J} \text{maint}_j x_j \leq M.$$

Capacity Constraints For each room type j, the number of patients served per year cannot exceed the total capacity provided by the constructed rooms:

$$y_j \leq \text{capacity}_j x_j \qquad \forall j \in J.$$

Demand Constraints Even if sufficient capacity exists, patient throughput is limited by market demand:

$$y_j \leq \text{demand}_j \qquad \forall j \in J.$$

Nonnegativity and Integrality All decision variables must be nonnegative:

$$x_j \geq 0, \quad y_j \geq 0 \qquad \forall j \in J.$$

Because rooms are indivisible assets, we additionally require

$$x_j \in \mathbb{Z} \qquad \forall j \in J.$$

3.5 Discussion

Together, the objective function and constraints define a mixed-integer linear program that captures the key economic and operational trade-offs in hospital construction planning. The formulation ensures feasibility with respect to budgets, capacity, and demand, while identifying the room mix that maximizes long-run annual net profit.

4 Break-Even Analysis

When does the hospital break even on its fixed investment, i.e. how many years of operation it takes for cumulative operating profit to equal the initial fixed cost?

$$\text{AnnualOperatingProfit} = \sum_{j \in J} p_j y_j^* - \sum_{j \in J} \text{maint}_j x_j^*.$$

$$\text{FixedCost} = \sum_{j \in J} (\text{build}_j + \text{machine}_j)\, x_j^*.$$

4.1 Compute Break-Even Year $T_{\text{break-even}}$

Assume the annual operating profit is constant over time (ignoring the time value of money) and the annual operating profit is positive since the first year, the number of years needed to recover the fixed investment (in other words break even) is:

$$T_{\text{break-even}} = \frac{\text{FixedCost}}{\text{AnnualOperatingProfit}} = \frac{\sum_{j \in J} (\text{build}_j + \text{machine}_j)\, x_j^*}{\sum_{j \in J} p_j y_j^* - \sum_{j \in J} \text{maint}_j x_j^*},$$

5 LP Implementation

5.1 LP Construction Using Gurobi

Our LP was implemented using Gurobi. Decision variables were defined as:

- Integer variables x_j for room construction
- Continuous variables y_j for patients served

The objective function maximized annualized net profit:

$$\max \sum_j p_j y_j - \sum_j \text{maint}_j x_j - \frac{1}{H} \sum_j (\text{build}_j + \text{machine}_j) x_j$$

This formulation explicitly balances high-revenue rooms against their long-run operating and capital costs.

Construction and maintenance budget constraints are enforced directly using linear inequalities, while capacity and demand constraints link patient throughput to room availability.

5.2 Greedy Benchmark Algorithm

To benchmark the LP, a greedy algorithm was implemented that incrementally adds rooms. At each step, the algorithm:

1. Computed the marginal profit increase ΔZ_j from adding one room of type j
2. Accounted for remaining patient demand and capacity
3. Checked feasibility with respect to remaining construction and maintenance budgets
4. Added the room with the highest positive ΔZ_j

This process repeated until no feasible room yielded positive marginal profit. Unlike the LP, the greedy method makes locally optimal decisions and does not consider global trade-offs across room types.
Below we have included a picture of our full LP and greedy algorithm implementation.



5.3 Scenario Analysis

We evaluate the performance of the LP model and the greedy benchmark under several scenarios designed to stress different economic and operational aspects of hospital construction. Each scenario modifies a subset of parameters to reflect realistic changes in budget availability, capital intensity, or service profitability. For each case, we compare room allocations, objective values, and relative optimality gaps.

5.3.1 Baseline Scenario

In the baseline scenario, construction costs, maintenance costs, and profit-per-patient values are well-balanced across all room types. Under these conditions, both the LP and greedy algorithms produce identical room allocations and objective values, resulting in a zero optimality gap.

This outcome indicates that when the problem structure is relatively separable and no strong trade-offs exist across room types, a local marginal-gain heuristic can recover the global optimum. Importantly, this scenario serves as a validation of the LP implementation and establishes a reference point for subsequent stress tests.

```
================== OPTIMAL LP SOLUTION ==================
Room counts (x_j) and patients served (y_j):

OR         rooms:   10.0   patients/yr:   7000.0
ICU        rooms:    8.0   patients/yr:   4000.0
WARD       rooms:   13.0   patients/yr:  15000.0
IMAGING    rooms:    6.0   patients/yr:  30000.0
CLINIC     rooms:    5.0   patients/yr:  20000.0

----------------- Profit and Break-even -----------------
Optimal annual net profit Z* (M$/yr):    55.4100
Annual operating profit (no fixed amortization) (M$/yr):    67.5000
Total fixed construction + equipment cost (M$):   120.9000
Break-even time (years):      1.79

================== GREEDY BENCHMARK ==================
Greedy room counts (x_G) and patients served (y_G):

OR         rooms:   10.0   patients/yr:   7000.0
ICU        rooms:    8.0   patients/yr:   4000.0
WARD       rooms:   13.0   patients/yr:  15000.0
IMAGING    rooms:    6.0   patients/yr:  30000.0
CLINIC     rooms:    5.0   patients/yr:  20000.0

----------------- Greedy Profit and Break-even -----------------
Greedy annual net profit Z_G (M$/yr):    55.4100
Greedy annual operating profit (M$/yr):    67.5000
Greedy fixed construction + equipment cost (M$):   120.9000
Greedy break-even time (years):      1.79

================== LP vs GREEDY COMPARISON ==================
Room counts (LP vs Greedy):
OR         LP rooms:    10.0   Greedy rooms:   10.0
ICU        LP rooms:     8.0   Greedy rooms:    8.0
WARD       LP rooms:    13.0   Greedy rooms:   13.0
IMAGING    LP rooms:     6.0   Greedy rooms:    6.0
CLINIC     LP rooms:     5.0   Greedy rooms:    5.0

Objective values (annual net profit, M$/yr):
LP optimal Z*:    55.4100
Greedy Z_G  :    55.4100
Relative optimality gap (LP vs. Greedy):  0.00%

In [13]:
```

Figure 1: Baseline scenario: LP and greedy solutions yield identical room allocations and objective values.

5.3.2 Scenario 1: Reduced Construction Budget

In Scenario 1, the total construction and equipment budget B is significantly reduced, representing a capital-constrained hospital expansion environment. Under this tighter budget, the LP solution reallocates investment toward room types with the highest long-run profit per dollar of capital expenditure, often favoring smaller or less capital-intensive services.

In contrast, the greedy algorithm selects rooms based on immediate marginal profit gains. This approach leads it to allocate capital to a suboptimal mix of room types, exhausting the budget before achieving the globally optimal configuration. As a result, the greedy solution exhibits a substantial optimality gap relative to the LP.

This scenario highlights the importance of global optimization when capital scarcity forces trade-offs between a small number of expensive rooms and a larger portfolio of moderately profitable ones.

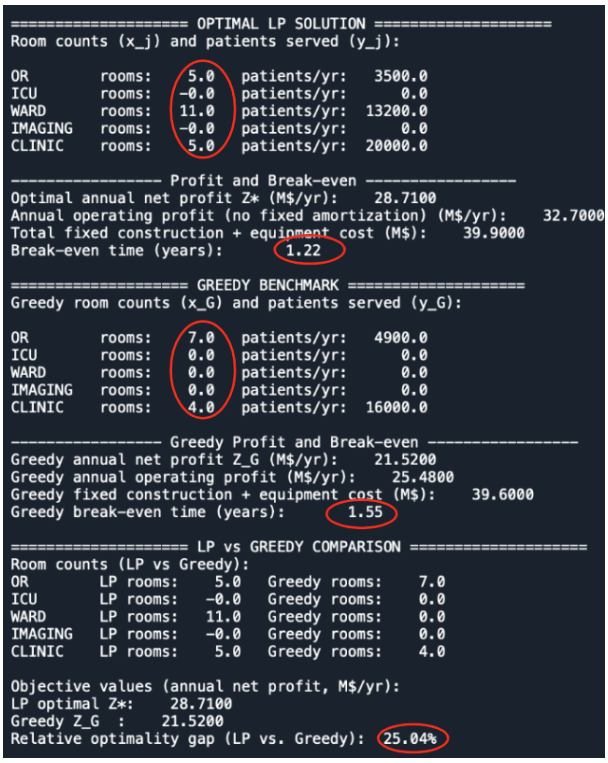


Figure 2: Scenario 1: Reduced construction budget leads to divergence between LP and greedy solutions.

5.3.3 Scenario 2: Imaging Becomes Very Capital-Intensive Under a Moderate Budget

In Scenario 2, we increased the *capital cost* of the IMAGING room type by raising both its construction cost and its equipment cost in the data dictionaries. We also set the construction/equipment budget to $B = 60$ (with $M = 25$ and planning horizon $H = 10$), creating a setting where capital must be allocated carefully across room types.

Even though IMAGING is excluded by *both* methods (0 rooms in both solutions), the scenario is still informative: increasing IMAGING capital cost and tightening B amplifies

trade-offs among OR/WARD/CLINIC. The LP uses the budget to build a more balanced mix (notably **more WARD capacity**), whereas the greedy method concentrates too early on OR additions and ends up with substantially fewer WARD rooms, leaving profitable ward demand under-served and lowering total profit.

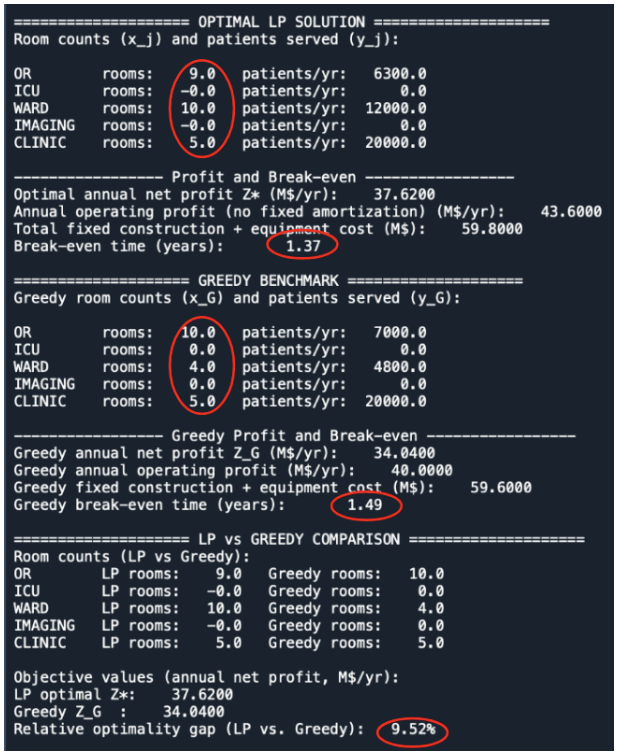


Figure 3: Scenario 2: Increased imaging capital costs eliminate imaging rooms from both LP and greedy solutions.

5.3.4 Scenario 3: Lower Imaging Profitability and Tighter Construction Budget

In Scenario 3, we made IMAGING less attractive by (i) keeping IMAGING *capital cost high* and (ii) *reducing IMAGING profit per patient*. We also tightened the construction/equipment budget further to $B = 55$ (with $M = 25$ and $H = 10$).

As in Scenario 2, IMAGING is excluded by both solutions (0 rooms) because its economics were worsened directly via higher capital cost and reduced per-patient profit. The main insight here is the *much larger divergence* between LP and greedy under a tighter budget: the LP invests in a diversified set of services (including substantial **WARD capacity** and some **ICU**), enabling it to serve multiple demand streams simultaneously and achieve far higher profit. The greedy approach, driven by step-by-step marginal gains, concentrates construction into ORs early and fails to build WARD/ICU capacity, leaving large amounts of demand unmet and dramatically reducing total net profit.

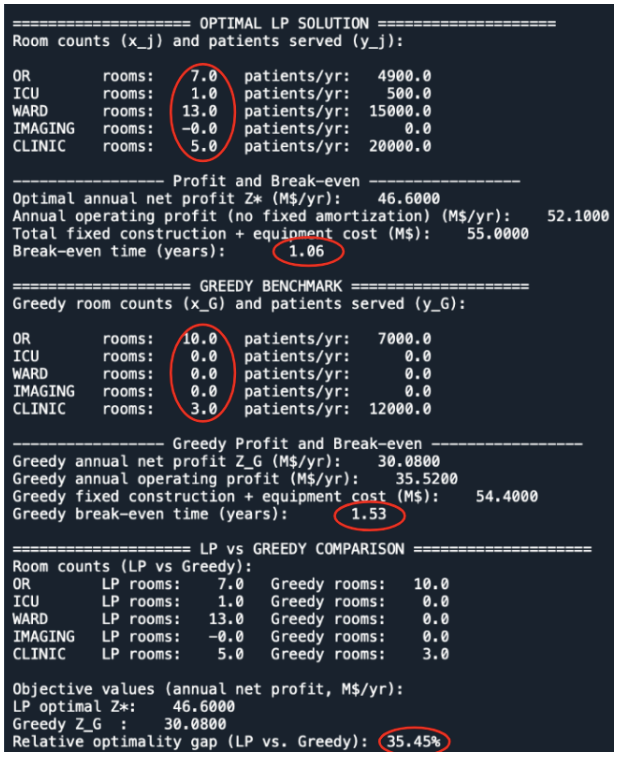


Figure 4: Scenario 3: Reduced imaging profitability causes significant divergence between LP and greedy solutions.

6 Conclusion

This project demonstrates the effectiveness of linear programming for hospital construction and capacity planning under realistic financial and operational constraints. The LP consistently identifies globally optimal room allocations by simultaneously accounting for budgets, demand, capacity, and long-run profitability.

Comparisons with a greedy benchmark reveal that while heuristic approaches may perform well under balanced conditions, they fail under tighter budgets or skewed cost structures. In such cases, local marginal decisions obscure important global trade-offs, leading to significant profit losses.

Overall, the LP framework provides a robust, interpretable, and economically grounded tool for strategic hospital planning, while the greedy comparison highlights the necessity of optimization-based decision-making in complex infrastructure systems.

7 Weaknesses & Next Steps

7.1 Weaknesses

- **Constant marginal cost and per-patient profit** We assume the hospital generates the same profit from each patient treated and disregard any economies of scale or inefficiency. For instance, when throughput increases, fixed cost such as equipment purchase can be further amortized, leading to lower cost (economies of sacle). On the other hand, during peak demand hours, higher-than-average overtime staff salary or resource fatigue would cause marginal cost to rise. The simulated data could underestimate the complexity of cost dynamic in hospital operation settings.

- **Constant room capacity without considering staffing and scheduling availability** In our model, patient throughput is upper-bounded by room availability and individual room's capacity. However, the number of staffs in active shifts and staff schedules could further upper-bound the amount of care that can be delivered based on given infrastructure.

- **One-to-one procedures-room mapping** To simplify the cost and revenue attribution in our model, we assume every procedure involves exactly one room type. This ignores different procedures that share equipment or one procedure utilizing more than one room/resource. Therefore, the patient throughput we optimized for may not be achieved in real world scenarios given the fixed room capacity and more complicated resource allocation.

- **Absence of cash flow discounting in break-even analysis** In real world, healthcare capital investment faces economic pressure from rising operating cost due to staff salary inflation, equipment depreciation, and slow insurance payment delaying profit realization, making our result optimistic. Further works could consider multi-period model and discounted cash flow to assess the break-even time for a more grounded investor expectation.

7.2 Next Steps

Demand for hospital services may vary over time, suggesting that room capacity planning could be extended to a multi-period setting rather than assuming a single representative year. In addition, rooms and medical equipment depreciate and require periodic replacement or upgrades, which could be incorporated through explicit renewal and refurbishment costs. Instead of annualizing fixed construction costs, a more detailed financial representation could

be achieved using discounted cash flow methods to track costs and revenues over multiple years. Finally, the model could be extended to allow staged construction, enabling the hospital to expand capacity gradually in response to evolving demand and budget conditions rather than committing all investment upfront.