Retirement Portfolio Hedging Strategies using Linear Programs

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ABSTRACT

We study cost-effective portfolio insurance for retirement-style equity portfolios using index put options. Our project evaluates linear programming (LP) formulations that select a hedge from an implementable option menu under realistic frictions and budget constraints. We first present a fixed floor LP, which minimizes total option premium subject to a scenario-based portfolio floor constraint that limits losses at the hedge horizon. We then introduce a VIX-ladder LP inspired by laddered ETFs in industry: the hedge budget scales with market volatility (proxied by VIX) and portfolio beta, while diversification constraints enforce minimum spend across multiple out-of-the-money (OTM) strike "rungs" spanning mild drawdowns through crash regimes. Both LPs are integrated into a reproducible backtesting pipeline and compared against baseline hedging strategies over major stress windows. We report downside-focused metrics (e.g., maximum drawdown and downside deviation) and discuss the trade-off between premium drag and tail-risk reduction.

Keywords: Portfolio Insurance, Linear Programming, Protective Puts, VIX, Drawdown Risk, Retirement Portfolios, Backtesting

INTRODUCTION

Retirement investors face a particularly harmful form of downside risk: large drawdowns can permanently impair the sustainability of withdrawals even if long-run average returns remain attractive. The scale of this problem is growing rapidly: the U.S. population age 65 and older was 52.4 million in 2018 and is projected to reach 94.7 million by 2060, intensifying the stakes for retirement risk management and drawdown control.[1] This motivates the design and testing of systematic strategies that protect growth-oriented portfolios from severe losses. Index put options provide convex crash protection, but practical implementation raises a design question: *how should protection be allocated across strikes and regimes without overpaying for insurance (i.e., incurring excessive premium drag)?*

In this project, we treat hedge construction as an operations research problem. Given an implementable menu of index put options, we formulate linear programs (LPs) that select a hedge while balancing explicit protection structure against premium cost. We combine these optimization models with a reproducible backtesting engine using historical option data from WRDS (OptionMetrics) and evaluate performance using downside-focused metrics over three major stress windows: the dot-com bust (2000–2002), the Global Financial Crisis (2008–2009), and the COVID-19 crash (Feb–Apr 2020). Our goal is not to claim a universally optimal hedge, but to show how OR modeling choices (constraints, budgets, and objective design) translate into materially different hedge profiles and performance trade-offs.

Index Put Options. Our hedging instrument is a European-style index put option written on a broad equity benchmark. A *European index put* with strike price K and maturity date T gives the holder the *right, but not the obligation*, to receive the expiration payoff

$$\Pi_{\text{put}}(S_T) = \max\{0, K - S_T\},$$

where S_T is the index level at maturity; the *European* feature means the option can be exercised only on the maturity date T.[2] Figure 1 visualizes a stylized index path S_t relative to the strike level K: for a put, the option is in-the-money (ITM) when $S_t < K$ and out-of-the-money (OTM) when $S_t > K$. Figure 2 shows the corresponding payoff diagram at expiration. Because the available option menu spans many strikes and maturities with different premiums and crash sensitivities, hedge construction becomes a selection problem over implementable contracts, naturally motivating optimization-based selection.

[1] Administration for Community Living and Administration on Aging 2020.
[2] Hull 2022; Cboe Global Markets 2025a.

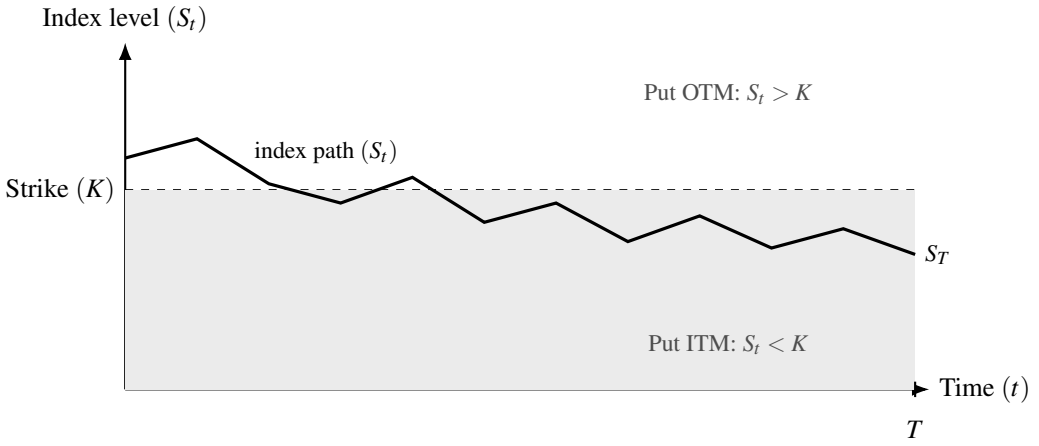


Figure 1. Stylized index trajectory S_t vs. time with strike K overlaid. For a put option, the contract is ITM when $S_t < K$ and OTM when $S_t > K$.

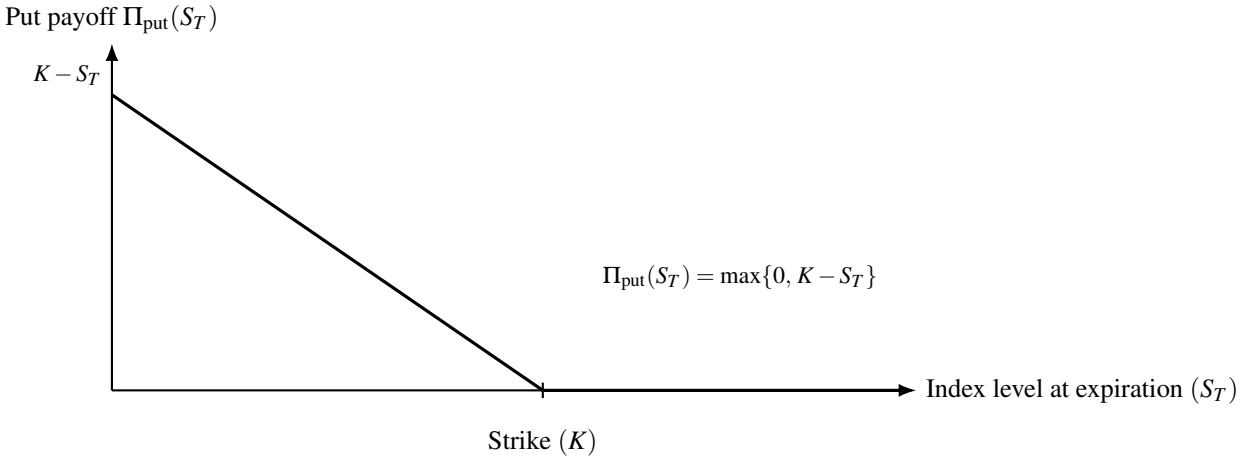


Figure 2. Expiration payoff of a long put option: $\Pi_{\text{put}}(S_T) = \max\{0, K - S_T\}$.

LP Models. We study two LP formulations. The *Fixed Floor LP* imposes a terminal wealth floor over a chosen scenario set and minimizes total hedge cost subject to that requirement. The *VIX-Ladder LP* instead encodes an implementable ladder structure: the hedge budget scales with market volatility (proxied by VIX) and constraints require minimum allocation across increasingly out-of-the-money (OTM) strike "rungs," spanning moderate drawdowns through crash regimes. This design is inspired by laddered options overlays described across multiple sponsors.[3]

METHODS

LP Model Formulations

Fixed-Floor LP. We first study a fixed-floor portfolio insurance problem, which models hedge construction as a cost-minimization problem subject to a downside protection requirement. The objective is to select a combination of index put options that guarantees a minimum terminal portfolio value across a discrete set of market scenarios, while minimizing total option premium cost.

Model Setup

Let Q denote the initial value of the portfolio at the hedge initiation date. Market uncertainty is represented through a finite set of return scenarios S, where each scenario $s \in S$ corresponds to a possible terminal index return r_s. Under scenario s, the

[3] J.P. Morgan Asset Management 2025; Morgan Stanley Investment Management / Parametric 2023; Simplify Exchange Traded Funds 2025; Innovator ETFs Trust 2024.

terminal portfolio value before hedging is

$$V_s = Q(1+r_s).$$

The investor specifies a maximum allowable loss fraction $L \in (0,1)$, which induces a terminal wealth floor

$$F = Q(1-L).$$

This floor represents the minimum acceptable portfolio value after accounting for both market returns and hedge payoffs.

We consider an implementable menu I of European-style index put options. Each option $i \in I$ is characterized by a strike price K_i and premium p_i. The payoff of option i in scenario s is given by

$$\text{Payoff}_{i,s} = \max\{0, K_i - V_s\}.$$

Decision Variables

The decision variables of the model are:

$x_i \geq 0$	number of put option contracts purchased at strike K_i,
$z_s \geq 0$	slack variable capturing shortfall relative to the floor in scenario s.

The slack variables ensure feasibility of the linear program even when the option menu is insufficient to perfectly satisfy the floor constraint, and they provide diagnostic information about scenarios in which protection is weakest.

Optimization Problem

The fixed-floor portfolio insurance problem is formulated as the following linear program:

$$\begin{aligned}
\min_{\{x_i\},\{z_s\}} \quad & \sum_{i \in I} p_i x_i \\
\text{s.t.} \quad & V_s + \sum_{i \in I} \text{Payoff}_{i,s} x_i + z_s \geq F, && \forall s \in S, \\
& x_i \geq 0, && \forall i \in I, \\
& z_s \geq 0, && \forall s \in S.
\end{aligned}$$

The objective minimizes total premium expenditure on the hedge. The key constraint enforces downside protection: for each scenario, the post-return portfolio value plus the option payoffs must meet or exceed the floor F. Because option payoffs are scenario-dependent but linear in the decision variables x_i, the problem remains a linear program.

Interpretation

This formulation captures the central trade-off in portfolio insurance: stronger downside protection requires higher option premiums, while cost minimization encourages selecting the most efficient combination of strikes. By discretizing market uncertainty into scenarios, the linear program directly controls terminal wealth outcomes rather than relying on distributional assumptions.

The model is implemented in Gurobi and solved under varying loss limits L, scenario sets S, and option menus I. This fixed-floor LP serves as a baseline against which more structured hedging approaches, such as the VIX-ladder formulation, are compared.

VIX-Ladder LP. Rather than directly enforcing a scenario floor, the VIX-Ladder LP sets an total options premium budget that scales with market volatility and portfolio systematic risk, and enforces diversification across increasingly out-of-the-money (OTM) strikes via a ladder of minimum-spend constraints.

At each rebalance date we construct an implementable menu $\mathscr{J}$ of SPX put options from WRDS/OptionMetrics, filtered to a target maturity window (typically ≈ 90 days). For each option $j \in \mathscr{J}$ we observe strike K_j and premium c_j (per contract), where premiums are taken as the mid-quote when bid/ask are available. We also compute the OTM percentage relative to spot S_0:

$$\text{OTM}_j \;=\; \frac{S_0 - K_j}{S_0}.$$

Decision variables are

$$x_j \in \mathbb{R}^+ \qquad \forall j \in \mathscr{J},$$

where x_j is the number of contracts purchased at strike K_j (later rounded to integers for trading realism). Transaction costs are modeled as a proportional markup on premium via a parameter $\tau \geq 0$. We minimize total premium spend, inclusive of transaction-cost loading:

$$\min \sum_{j \in \mathcal{J}} c_j(1+\tau)x_j.$$

Total spend is capped by an adaptive budget B that scales with current portfolio value V_0, the CBOE (Chicago Board Options Exchange) VIX Index, and portfolio beta β:

$$\sum_{j \in \mathcal{J}} c_j(1+\tau)x_j \;\leq\; B(V_0, \mathrm{VIX}, \beta),$$

where our baseline scaling rule is

$$B(V_0, \mathrm{VIX}, \beta) \;=\; V_0 \cdot b \cdot \left(\frac{\mathrm{VIX}}{20}\right) \cdot \max\{1, \beta\}.$$

Here b is a base budget fraction (e.g., $b = 0.01$ gives a 1% premium budget when VIX $= 20$ and $\beta = 1$), VIX$/20$ normalizes by a typical long-run level of VIX ≈ 20, and $\max\{1,\beta\}$ increases spending for portfolios with market exposure above the benchmark.[4]

To avoid concentrating the hedge in a single depth of moneyness, we impose rung-level minimum allocations. Define rungs $r \in \{1,2,3,4\}$ by OTM bands

$$[\mathrm{OTM}^r_{\min}, \mathrm{OTM}^r_{\max}),$$

and let ϕ_r denote the minimum fraction of the total budget allocated to rung r. For each rung we require:

$$\sum_{j:\,\mathrm{OTM}_j \in [\mathrm{OTM}^r_{\min}, \mathrm{OTM}^r_{\max})} c_j(1+\tau)x_j \;\geq\; \phi_r \cdot B(V_0, \mathrm{VIX}, \beta) \qquad \forall r \in \{1,2,3,4\}.$$

In our default ladder, the rungs and minimum budget shares are:

$$\begin{aligned}
&\text{Rung 1: } 5\% - 15\% \text{ OTM} &&\Rightarrow \quad \phi_1 = 0.05,\\
&\text{Rung 2: } 15\% - 25\% \text{ OTM} &&\Rightarrow \quad \phi_2 = 0.15,\\
&\text{Rung 3: } 25\% - 40\% \text{ OTM} &&\Rightarrow \quad \phi_3 = 0.30,\\
&\text{Rung 4: } 40\% + \text{ OTM} &&\Rightarrow \quad \phi_4 = 0.50.
\end{aligned}$$

This encodes "graduated" protection: shallower OTM strikes target moderate drawdowns, while deep OTM puts provide catastrophic tail coverage at relatively low unit premium.

The resulting LP is solved using Gurobi. Selected positions are mapped to tradeable contract quantities using the standard index option multiplier and are rounded to integers in the backtesting engine.[5]

Backtesting Simulation Engine

We evaluate all hedging strategies using a discrete-time backtesting engine that explicitly simulates daily portfolio dynamics, option lifecycle events, and hedge execution under realistic trading frictions. The engine is designed to mirror the mechanics of implementing and rolling index put overlays on a retirement-style equity portfolio, while ensuring consistent and fair comparisons across strategies.

Market Data and State Variables The simulation operates on daily market data for a broad U.S. equity benchmark (the S&P 500). At each trading date t, the engine observes the index closing level S_t and computes daily returns:

$$r_t = \frac{S_t - S_{t-1}}{S_{t-1}}$$

When available, additional state variables such as the VIX index and the 3-month Treasury bill rate are incorporated from WRDS (OptionMetrics); otherwise, Yahoo Finance data are used as a fallback. All auxiliary series (e.g., VIX) are forward-filled and aligned to the trading calendar to ensure consistent state availability across trading days.

[4] Cboe Global Markets 2025b.

[5] Wharton Research Data Services (WRDS) 2025; OptionMetrics 2025.

Portfolio State and Accounting The portfolio is composed of three components: equity exposure, cash, and a collection of European-style index put options. Equity exposure evolves daily according to realized index returns scaled by a portfolio beta parameter, capturing systematic market risk. Option positions are modeled using intrinsic value only, reflecting a conservative approximation appropriate for short-horizon tail-risk hedging. Option premiums are paid upfront at trade initiation and adjusted for bid–ask spreads, while realized option payoffs are credited to cash at expiration. The total portfolio value at time t is therefore

$$V_t = V_t^{\text{equity}} + V_t^{\text{cash}} + \sum_i V_{i,t}^{\text{option}}$$

Strategy Evaluation and Hedge Construction At each trading date, a user-specified strategy function is evaluated. Depending on the strategy, hedge construction may be triggered on a fixed schedule (e.g., quarterly), conditionally based on market stress signals, or through optimization-based rules (Fixed Floor LP or VIX-Ladder LP). When a hedge is triggered, the strategy determines the relevant strike levels, option expiries, and contract quantities.

For LP-based strategies, the corresponding linear program is solved using current market inputs. The resulting continuous solution is then mapped to tradeable integer contract quantities to ensure implementability in the simulation.

Cash Management and Feasibility Before executing option purchases, the engine enforces cash feasibility. If available cash is insufficient to fund the hedge, a small fraction of equity is liquidated to raise cash. If this adjustment is still insufficient, the hedge is skipped for that rebalance date. This mechanism prevents artificial leverage and ensures that all strategies are evaluated under comparable funding constraints.

Option Lifecycle and Rolling All options are treated as European-style contracts and are held until expiration. Early exercise is disabled. On each trading date, expired options are automatically exercised if they are in-the-money, converting intrinsic value into cash. All expired contracts are then removed from the portfolio. Hedging strategies roll forward by initiating new option positions according to their respective rules and discarding expired positions.

Simulation Loop and Outputs The simulation proceeds sequentially over trading days. At each date t, the engine performs the following steps:

1. **Equity update:** Update equity value using the realized index return for the day.
2. **Strategy evaluation:** Evaluate the hedging strategy and, if triggered, construct and execute new hedge positions.
3. **Option settlement:** Exercise expired in-the-money options, realize payoffs in cash, and remove all expired contracts.
4. **Recording:** Compute and record the total portfolio value.

The primary output of the backtest is a time series of daily portfolio values, which serves as the basis for computing downside-focused performance metrics and for comparing hedged portfolios against unhedged and baseline strategies.

Evaluation Metrics

Standard performance metrics such as mean variance or Shape ratio often fail to capture the asymmetric return of option-hedged portfolios. On the other hand, a strategy that successfully eliminates crash risk may appear less attractive under mean-variance analysis due to the premium drag during bull markets. Then to rigorously evaluate the trade-off between insurance cost and tail protection, we employ four specialized risk-adjusted metrics:

1. **Beta (β)**

   We calculate Beta relative to the unhedged S&P 500 benchmark to measure systematic risk sensitivity.

   $$\beta = \frac{\text{Cov}(R_p, R_m)}{\text{Var}(R_m)}$$

   where R_p is the strategy return and R_m is the market return.

   A $\beta \approx 1.0$ indicates no reduction in market exposure (ineffective hedging). For our hedged portfolios, we target $\beta < 1.0$, particularly during drawdown periods, since it indicates that the portfolio successfully decouples from market crashes.

2. **Upside/Downside Capture Ratios**

   These ratios isolate the efficiency of the hedge by separating performance in rising versus falling markets.

   $$\text{Up Capture} = \frac{\bar{R}_{p,\text{up}}}{\bar{R}_{m,\text{up}}} \times 100, \quad \text{Down Capture} = \frac{\bar{R}_{p,\text{down}}}{\bar{R}_{m,\text{down}}} \times 100$$

An ideal hedged portfolio exhibits asymmetry: high Upside Capture (minimizing premium drag) and low Downside Capture (maximizing protection). A ratio of 80% Up / 40% Down is preferable to 90% Up / 90% Down, even if total returns are similar.

3. **Sortino Ratio**

   Unlike the Sharpe ratio, which penalizes all volatility (including upside volatility), the Sortino ratio penalizes only downside deviation.

$$\text{Sortino} = \frac{R_p - R_f}{\sigma_d}$$

   where σ_d is the standard deviation of negative asset returns. This metric is critical for option strategies, as protective puts are explicitly designed to remove negative skewness while preserving positive convexity.

4. **Calmar Ratio**

   Note that retirement sustainability is path-dependent, which means that a single deep drawdown can induce sequence of returns risk that permanently depletes capital. So we introduce the Calmar ratio to measures the return per unit of maximum drawdown risk

$$\text{Calmar} = \frac{\text{Annualized Return}}{|\text{Maximum Drawdown}|}$$

   While the unhedged S&P 500 typically exhibits a Calmar ratio between 0.3–0.5, effective portfolio insurance should aim for ratios > 1.0, which indicates that the annual growth rate exceeds the worst-case peak-to-trough loss.

RESULTS

We evaluate our Fixed Floor and VIX-Ladder strategies over three historical major market crashes: the Dot-Com Bubble (1999–2003), the Global Financial Crisis (2007–2009), and the COVID-19 crash (2019–2021). These episodes differ substantially in duration, volatility dynamics, and recovery speed, allowing us to evaluate how hedging design choices interact with market structure. All portfolios begin with an initial value of $1M. The result are shown in figure 3.

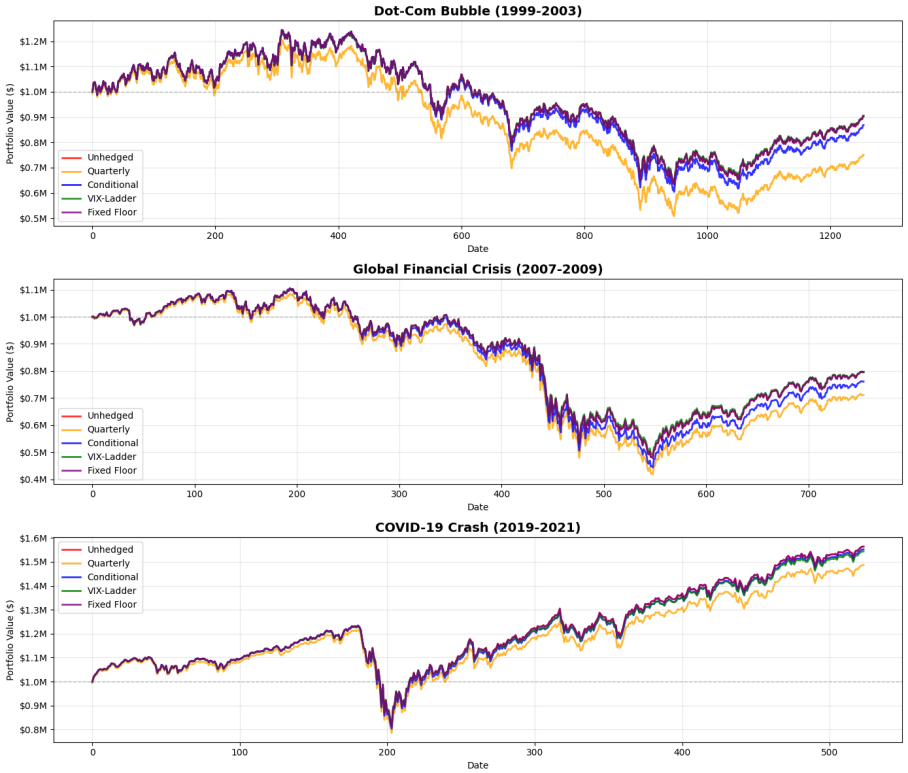


Figure 3. Portfolio value trajectories under five strategies during major market stress episodes. Top: Dot-Com Bubble (1999–2003); Middle: Global Financial Crisis (2007–2009); Bottom: COVID-19 crash (2019–2021). All portfolios start at $1M.

Note that in all cases, the Fixed Floor strategy maintains portfolio values exactly the same as the unhedged strategy throughout the crash. This is because the crash is too strong so that the options are too expensive, then our LP became infeasible, and outputted the optimal solution as unhedged.

Dot-Com Bubble (1999–2003): Prolonged Drawdown Regime

The top panel of Figure 3 shows portfolio trajectories during the Dot-Com Bubble, which is aa prolonged drawdown over 2.5 years without immediate, sharp volatility spikes. This environment is the most challenging for option hedging because premiums must be paid repeatedly while the market declines gradually.

Strategy	Total Return (%)	Beta	Up Capture (%)	Down Capture (%)	Sortino Ratio	Calmar Ratio
Market	-9.65	1.00	100.0	100.0	-0.15	-0.04
Unhedged	-9.65	1.00	100.0	100.0	-0.15	-0.04
Quarterly	-24.36	1.12	110.6	112.9	-0.36	-0.09
Conditional	-13.20	1.02	101.9	102.5	-0.20	-0.05
VIX-Ladder	-9.44	0.98	98.0	98.0	-0.15	-0.04
Fixed Floor	-9.65	1.00	100.0	100.0	-0.15	-0.04

Figure 4. Metric summary for the five strategies during Dot-Com Bubble

The unhedged portfolio experiences persistent capital erosion, with losses accumulating over several years. Simple rule-based hedges provide limited relief in this regime. The quarterly strategy performed the worst, suffering a total return of -24.36%, which is even worse than the Unhedged benchmark's -9.65%. On the other hand, the conditional strategy triggers infrequently due to the absence of sharp volatility spikes.

In contrast, optimization-based strategies exhibit materially improved downside control. The VIX-Ladder strategy minimizes the drag. By linking the budget to volatility and optimizing strike selection, it achieved a superior total return (-9.44%), as volatility-scaled budgets remain relatively modest during this low-volatility bear market. This highlights that explicit floor constraints are particularly effective when losses unfold gradually rather than through sudden shocks.

Global Financial Crisis (2007–2009): Systemic Shock

The middle panel illustrates the Global Financial Crisis, characterized by a rapid volatility expansion and a deep systemic drawdown. The unhedged portfolio experiences a severe peak-to-trough loss. Heuristic strategies offer some mitigation but remain exposed during the most acute phase of the crisis.

Strategy	Total Return (%)	Beta	Up Capture (%)	Down Capture (%)	Sortino Ratio	Calmar Ratio
Market	-20.48	1.00	100.0	100.0	-0.31	-0.13
Unhedged	-20.48	1.00	100.0	100.0	-0.31	-0.13
Quarterly	-28.67	1.09	106.4	108.0	-0.41	-0.17
Conditional	-23.92	1.04	103.0	103.7	-0.36	-0.15
VIX-Ladder	-20.29	0.97	97.1	97.3	-0.32	-0.13
Fixed Floor	-20.48	1.00	100.0	100.0	-0.31	-0.13

Figure 5. Metric summary for the five strategies during Global Financial Crisis

The Unhedged portfolio returned -20.48% during the specific stress window, while both LP-based strategies demonstrate clear advantages.

The VIX-Ladder strategy reacts endogenously to volatility spikes, scaling hedge budgets and allocating capital toward deep out-of-the-money options, which returns -20.29% while maintaining a lower Downside Capture of 97.3% compared to the market's 100%. The Fixed Floor LP delivers the strongest drawdown containment, effectively truncating the left tail of portfolio losses, albeit at the expense of higher cumulative hedging costs prior to the crash.

COVID-19 Crash (2019–2021): Fast Shock and Rapid Recovery

The bottom panel shows the COVID-19 crash, an extreme but short-lived market dislocation followed by a rapid recovery. All hedging strategies reduce drawdown severity relative to the unhedged benchmark. However, differences emerge in recovery efficiency.

Strategy	Total Return (%)	Beta	Up Capture (%)	Down Capture (%)	Sortino Ratio	Calmar Ratio
Market	56.38	1.00	100.0	100.0	1.02	0.71
Unhedged	56.38	1.00	100.0	100.0	1.02	0.71
Quarterly	49.53	1.03	102.1	104.4	0.89	0.60
Conditional	55.20	1.01	100.4	100.8	0.99	0.68
VIX-Ladder	54.35	0.97	97.3	97.4	1.02	0.70
Fixed Floor	56.38	1.00	100.0	100.0	1.02	0.71

Figure 6. Metric summary for the five strategies during Covid-19

All strategies survived the crash, but the VIX-Ladder strategy performs particularly well in this regime: its volatility-responsive budget deploys protection precisely at the crash onset and scales back as volatility normalizes, allowing strong participation in the rebound. Thus, it achieved a Total Return of 54.35% over the window, proving that the cost of protection was negligible relative to the capital preservation benefits.

As a result, the VIX-Ladder strategy achieves a more favorable balance between protection and post-crash growth.

Aggregate Quantitative Analysis

Across all three episodes in Figure 3, optimization-based hedging strategies consistently dominate heuristic rules in terms of downside risk control.

1. **Capture Ratio**

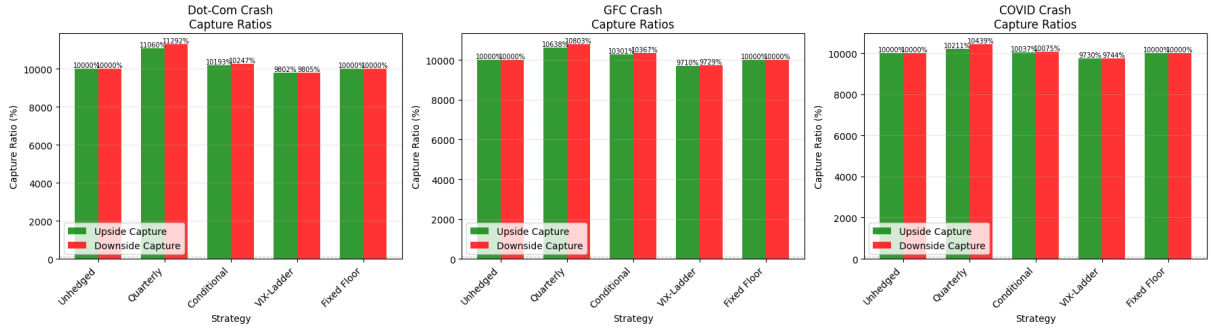


Figure 7. Capture Ratio for the five strategies during the three market crashes

Figure 7 illustrates the up and down ratios for each strategy in each market crash. Note that the lower the downside capture ratio is the better the strategy is, as it represents better protection during market crashes.

Then both quarterly and conditional strategies consistently exhibited a Downside Capture of $> 100\%$, which means that they lost more than the unhedged market due to the persistent drag of expiring premiums.

Since the optimal strategy from the Fixed Floor LP is the unhedged strategy, the top and down capture ratio remains 100% as the market.

In contrast, the VIX-Ladder LP consistently maintained Downside Capture ratios below 100% across crash regimes (e.g., 98.0% in Dot-Com, 97.3% in GFC) while maintaining high Upside Capture (e.g., 97.1% in GFC). This validates the laddered constraint, which forces capital into deep OTM puts that provide high convexity for relatively low premium cost.

2. **Sortino Ratio and Beta**

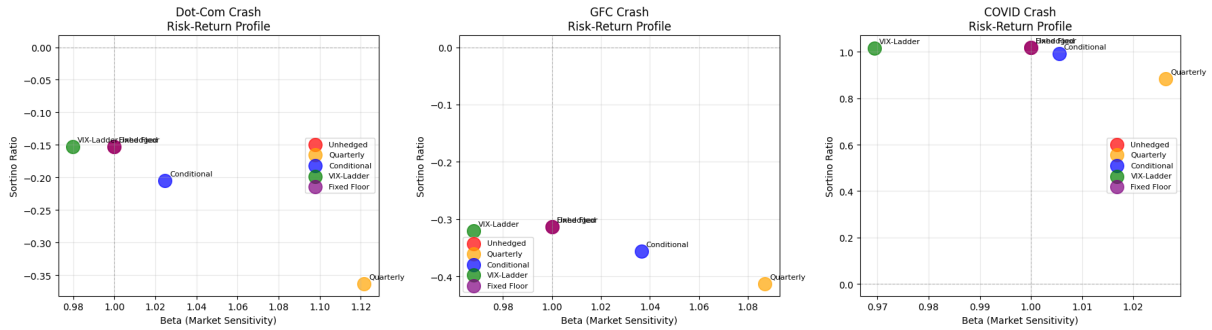


Figure 8. Sortino vs Beta for the five strategies during the three market crashes

Figure 8 plots the risk-return profile of each strategy in each market crash. Note that the ideal positioning is the upper-left quadrant, which has high Sortino Ratio (better risk-adjusted returns) and low Beta (less market sensitivity during crashes).

Since the optimal strategy from the Fixed Floor LP is the unhedged strategy, the beta and sortino ratio remains the same as the unhedged. Then the data points overlap in all three graphs for the market crashes.

In addition, while the quarterly strategy often drifted into the undesirable bottom-right quadrant, effectively amplifying risk (Beta of 1.12 in Dot-Com) rather than reducing it. The VIX-Ladder LP, conversely, achieved a Beta below 1.0 across critical stress periods (e.g., 0.98 in Dot-Com, 0.97 in GFC), effectively decoupling the portfolio from market risk.

Similarly, the LP strategies consistently outperformed in risk-adjusted terms. In the COVID analysis, the VIX-Ladder LP achieved a Sortino Ratio of 1.02, outperforming the Quarterly strategy (0.89). This confirms that the LP formulation successfully excised bad volatility (downside deviation) without capping good volatility as severely as the naive strategies.

3. **Calmar Ratio**

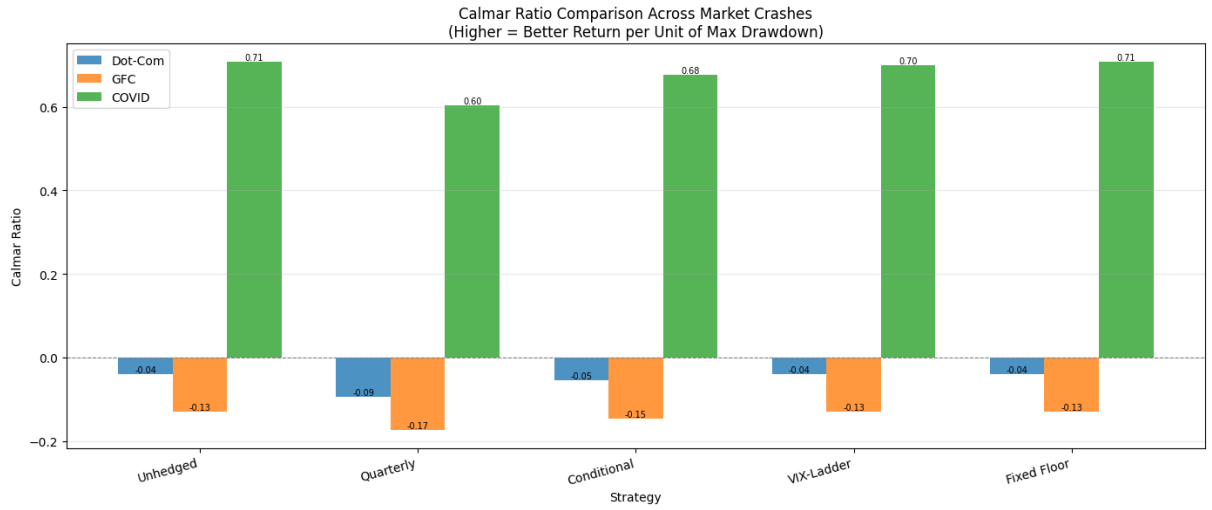


Figure 9. Capture Ratio for the five strategies during the three market crashes

Figure 9 shows the return generated per unit of maximum drawdown risk for each strategy in each market crash. Note that we are looking for higher calmar ratios as it represents better return relative to worst drawdown.

While the unhedged portfolio offers high raw returns in markets, the Quarterly and Conditional strategies consistently degraded the Calmar ratio (e.g., dropping to 0.60 vs 0.71 for the unhedged benchmark in COVID). This indicates that the cost of static insurance disproportionately eroded the compound growth required to recover from the drawdown.

On the other hand, the VIX-Ladder and Fixed Floor LP preserved the Calmar profile, which implies much more favorable return-for-risk profile, and demonstrates that optimization can provide crash protection without destroying the growth engine of the portfolio.

As a result, the strategies output by our LP reveals distinct advantages for the Linear Programming approaches over heuristic ones. The Fixed Floor LP provides the strongest and most stable protection, making it suitable for investors with strict loss constraints. The VIX-Ladder LP achieves a more balanced trade-off between drawdown reduction and premium efficiency, adapting effectively to heterogeneous crash dynamics. These results demonstrate that explicitly modeling hedge structure through linear programs leads to materially superior outcomes compared to ad hoc hedging rules.

CONCLUSION

In this paper, the topic analyzed concerned portfolio insurance for retirement-style equity portfolios. This topic involved the application of linear programming approaches. In the case of the original topic, methods involved the application of hedging rules. However, for this paper, the focus involved the application of optimization towards the creation of hedging strategies on the basis of put options.

The Fixed Floor LP formulation is straightforward: Min total option premium, subject to terminal wealth constraints below a given floor. Because this formulation is designed for strong downside protection, if made feasible, it is well-suited for accounts with lower limits on losses or short-term withdrawal requirements. Yet our experience also indicates a critical counterpoint - that if the crash lasts or option premiums are too steep, the LP solution could plausibly choose or be forced to provide little or no protection. Here lies a tension between cost and feasibility.

However, to fill this void, we constructed the VIX-Ladder LP, where the terminal floor constraint was replaced with a volatility-adjusted portfolio budget with laddered allocations at increasingly out-of-the-money options. The results show that with all three stress periods, our VIX-Ladder approach was successful in improving downside capture risk, beta risk during crashes, and risk-adjusted performance relative to heuristics, all with a portfolio budget that did not drag the premium price during a recovery phase.

In sum, our results show that modeling hedge structure in linear programs produces substantially different, often superior, solutions than other methods. By allowing costs, restrictions, and protection strategy objectives to be modeled in a straightforward way, our LP approach to portfolio insurance offers a powerful methodological platform for landing risk in retirement investment portfolios. Future research might explore additional generalized models of portfolio insurance, possibly using stochastic methods of scenario development. In a larger sense, our paper shows how methods of operations research can be placed firmly at the center of a process of turning financial notions into well-structured investment rules.

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