Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 3: Due Monday October 5.

Q1 Show that the expected number of proposals in the Gale-Shapley algorithm for Stable Marriage is $O(n \log n)$.

Q2 Let $\mathcal{T}$ denote the set of spanning trees of the complete graph K_n. Each tree in $\mathcal{T}$ can be represented by a 0/1 vector $\mathbf{x}_T \in \mathbb{R}^{n(n-1)/2}$. Let P denote the convex hull of $\{\mathbf{x}_T : T \in \mathcal{T}\}$. Describe P as the intersection of half-spaces.

Q3 How would you use Lagrangean Relaxation in order to solve the following optimization problem:

$$\begin{aligned} &\text{Maximize } \mathbf{c}^T\mathbf{x}_T. \\ &\text{Subject to } T \in \mathcal{T}. \\ &\mathbf{a}^T\mathbf{x}_T \le A, \end{aligned}$$

where $\mathbf{a} \in \mathbb{R}_+^N$, $N = \binom{n}{2}$ and $A \in \mathbb{R}_+$.