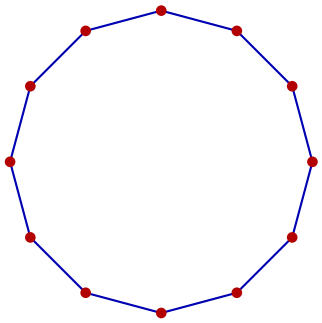
Department of Mathematical Sciences
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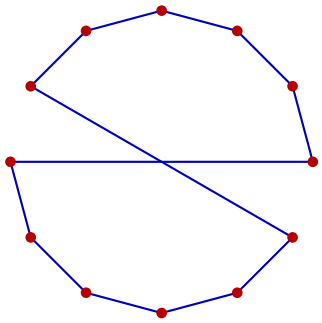
OPERATIONS RESEARCH II 21-393

Homework 2: Due Monday September 21.

Q1 The 2-OPT heuristic for the TSP can be described as follows: we are given costs $C(i,j)$ for the edge $\{i,j\}$. A *tour* corresponds to a permutation π of the set $[n]$. The cost of a tour $C(\pi) = \sum_{i=1}^{n} C(\pi(i), \pi(i+1))$ where $\pi(n+1) = \pi(1)$. The *2-swap(i,j)*(i,j,π) where $i < j$ replaces π by

$$\pi(1), \ldots, \pi(i), \pi(j), \pi(j-1), \ldots, \pi(i+1), \pi(j+1), \ldots, \pi(j+1), \ldots, \pi(n).$$





The 2-OPT algorithm performs 2-swaps so long as there is an i,j such that $C(2swap(i,j,\pi)) < C(\pi)$, in which case it will replace π by $2swap(i,j,\pi)$. The costs are defined as follows: an adversary chooses an initial $C, C(i,j) \leq 1$ and then nature adds a random value $\epsilon(i,j)$ to $C(i,j)$ where $\epsilon(i,j)$ is uniform in $[-\epsilon, \epsilon]$. Starting at an arbitrary π, bound the expected number of steps until the algorithm terminates, in terms of n and ϵ.

Q2 How would you adapt the simplex algorithm to solve the following LP:

$$\text{Minimise} \quad \sum_{1 \leq i < j \leq n} C(i,j) x_{i,j}.$$

$$\text{Subject to} \quad \sum_{i \in S, j \notin S} x_{i,j} \geq 1, \quad \text{for all } S \subseteq [n], S \neq \emptyset.$$

$$0 \leq x_{i,j} \leq 1, \quad \text{for all } i \neq j.$$

(The problem here is that there are more than 2^n constraints.)