Department of Mathematical Sciences
CARNEGIE MELLON UNIVERSITY

OPERATIONS RESEARCH II 21-393

Homework 1: Due Monday September 7.

Q1 Let $\mathbf{A}$ be an $m \times n$ matrix of 0's and 1's where $\mathbf{A}(i,j) = 1$ independently with probability p, where $0 < p < 1$ independent of m, n. Show that if $n \leq -(1-\epsilon)\frac{\log m}{\log(1-p)}$ then w.h.p. there is no solution to $\mathbf{Ax} \geq \mathbf{1}^m$ where $x_j \in \{0,1\}$ for $j = 1, 2, \ldots, n$.
[Hint: use the Chebyshev inequality to show that w.h.p. there will be a zero row in $\mathbf{A}$.]

Q2 Show that if $n \geq -(1+\epsilon)\frac{\log m}{\log(1-p)}$ then putting $\mathbf{x} = \mathbf{1}^n$ does satisfy $\mathbf{Ax} \geq \mathbf{1}^m$.
[Hint: use the first moment method to show that w.h.p. there are no uncovered RHS's.]

Q3 Suppose that we consider the set covering problem

$$\begin{aligned} &\text{Minimize } \mathbf{c}^T\mathbf{x} \\ &\text{Subject to } \mathbf{Ax} \geq \mathbf{1}^m, \\ &\qquad x_j = 0 \text{ or } 1, j = 1, 2, \ldots, n. \end{aligned}$$

Assume that $n \geq -(1+\epsilon)\frac{\log m}{\log(1-p)}$ and that $c_1, c_2, \ldots, c_n$ are independent uniform $[0,1]$ random variables. Estimate the cost of the minimum solution w.h.p.