# The Origin of Math Studies at CMU

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The mathematical infrastructure used in Math Studies is almost exclusively the one described in Section 3 of my essay *The Conceptual Infrastructure of Mathematics*, and we will call it the *Stage 3 Infrastructure*. Much of the instruction and literature on pure and applied mathematics used today is still based on the Stage 2 infrastructure as described in Section 2 of my essay. Here is an example of the distinction:

Stage 2: $x$ is the independent variable and $y$ is the dependent variable and they are related by the function $y = \sin(x^2)$ with derivative $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x\cos(x^2)$

Stage 3: Consider the function $f := \sin \circ \iota^2 : \mathbb{R} \longrightarrow \mathbb{R}$. Its derivative is $f^\bullet = 2\iota \cos \circ \iota^2 : \mathbb{R} \longrightarrow \mathbb{R}$. ($\iota$ is the identity mapping of the set $\mathbb{R}$ of real numbers.)

The shift from Stage 2 to Stage 3 infrastructure started in the late 19th century. The most important instigator for this shift was Georg Cantor (1846-1918). He analyzed the concept of a set. (He used the German terms *Mannigfaltigkeit* or *Menge*.) In essence, he proposed that concepts should be converted into sets. For example, instead of talking about natural numbers and real numbers as concepts, he thought it to be useful to consider the *set* $\mathbb{N}$ of all natural numbers and the *set* $\mathbb{R}$ of all real numbers. Doing so recklessly soon elicited objections and led to paradoxes. The objections to his work were occasionally fierce: Poincaré referred to Cantor's ideas as a "grave disease infecting the discipline of mathematics", and Kronecker's public opposition and personal attacks included describing Cantor as a "scientific charlatan", a "renegade" and a "corrupter of youth." Writing decades after Cantor's death, Wittgenstein lamented that mathematics is "ridden through and through with the pernicious idioms of set theory," which he dismissed as "utter nonsense" that is "laughable" and "wrong". Cantor's recurring bouts of depression from 1884 to the end of his life were once blamed on the hostile attitude of many of his contemporaries, but these episodes can now be seen as probable manifestations of a bipolar disorder.

The harsh criticism has been matched by later accolades. In 1904, the Royal Society awarded Cantor its Sylvester Medal, the highest honor it can confer. Cantor believed his theory of transfinite numbers had been communicated to him by God. David Hilbert defended it from its critics by famously declaring: "No one shall expel us from the Paradise that Cantor has created."

The beginning of the 20th century brought the introduction of abstract mathematical structures, such as groups, fields, and rings. The most important names involved were David Hilbert (1862-1943), Emmy Noether (1882-1935), and Emil Artin (1898-1962). These developments were systematically described in Bartel van der Waerden's *Moderne Algebra*, a two-volume monograph, published in 1930-1931, that forever changed for the mathematical world the meaning of the word *algebra* from the theory of equations to the theory of algebraic structures. (The book was translated into English with the title *Modern Algebra*.)

In 1934 a group of young French mathematicians, all connected to the École Normale Supérieure in Paris, got together to do for mathematics in general what Van der Waerden had done for algebra. Under the pseudonym *Nicolas Bourbaki*, the group produced, from 1938 to 1983, the series *Elements of Mathematics (Éléments de mathématique)* which contains the following volumes (with the original French titles in parentheses):

I Set theory (Théorie des ensembles)

II Algebra (Algèbre)

III Topology (Topologie générale)

IV Functions of one real variable (Fonctions d'une variable réelle)

V Topological vector spaces (Espaces vectoriels topologiques)

VI Integration (Intégration)

VII Commutative algebra (Algèbre commutative)

VIII Lie groups (Groupes et algèbres de Lie)

IX Spectral theory (Thóries spectrales)

The founding members of the Bourbaki group were Henri Cartan, Claude Chevalley, Jean Coulomb, Jean Delsarte, Jean Dieudonné, Charles Ehresmann, René de Possel, Szolem Mandelbrojt and André Weil,. Other notable participants in later days were Laurent Schwartz, Jean-Pierre Serre, Alexander Grothendieck, Samuel Eilenberg, Serge Lang, Roger Godement, and Armand Borel.

Recently, a very good book about Bourbaki, written by Maurice Mashaal, was published first in French and, in 2006, in English by the American Mathematica Society. The title is *Bourbaki, A Secret Society of Mathematicians*.

Now, the mathematical world is divided. Some people, including me, believe that Bourbaki provided the most important advance in mathematics in the 20th century. Others are more critical and offer objections, some almost as fierce as the objections in the 191h century to the work of Georg Cantor.

I graduated from high school in Germany in 1943. Even while still in high school I became acquainted with set theory by reading a small book by Erich Kamke entitled *Mengenlehre*, which describes Cantors ideas. (It has been translated into English and the translation, entitled *Set Theory*, is still available on Amazon.com.) I was fascinated by this book and even took it with me when I was drafted into the German armed forces in World War II. In the Spring of 1944 I audited a course on Modern Algebra at the University of Berlin: so I became familiar with this subject even before I formally enrolled as a student at the Technical University of Berlin in the Spring of 1946.

I spent the academic year 1949-50 at the University of Paris, France, with the help of a fellowship from the French government. It was there that I discovered Bourbaki. Even though my financial means were very limited, I bought all the books of the Bourbaki series published until that time. I studied them very carefully and became enthusiastic about the way Bourbaki treated mathematics.

I spent the academic year 1953-54 in the US as a graduate student at Indiana University in Bloomington. Shortly after arriving there, I took my Ph.D. qualifying exam. The people in the examination committee circulated a note saying “too much Bourbaki”. I received my Ph.D. in September 1954 and returned to Germany for a year. I immigrated to the US in the Fall of 1955 as an Instructor at the University of Southern California. In the Spring of 1956, I was offered a position of Associate Professor at Carnegie Tech (now CMU) with a 50% increase in salary and have been here ever since.

In 1968, Professor Juan Schäffer accepted a position as Professor at CMU, moving to Pittsburgh with his family from Montevideo, Uruguay. We both became friends and found that our attitudes about mathematics were very similar. Thus, we and some others proposed a 4-semester honors program for gifted undergraduate students. Our proposal was supported whole-heartedly be the then head of the mathematics department Ignace Kolodner. The term “Mathematical Studies” for this program was proposed by Professor Richard Moore. Both of us were involved in this program for many years, sometimes together and sometimes with others. We disliked the way mathematics was taught and done at that time and proposed a new way to present it as an integrated whole and to avoid its traditional division into separate and seemingly unrelated courses. We became a group of two, somewhat similar to the much bigger Bourbaki group, as described in the book by Maurice Mashaal: “Gradually, the group’s extensive reflections and lively discussions led to a new vision of mathematics, a modern way of teaching it and even doing it.” Also, our aim had a more limited scope than the one achieved by Bourbaki. We concentrated on the more elementary and more special parts of mathematics, for example on undergraduate calculus and analysis.

When appropriate, we followed the notation and terminology of Bourbaki. We developed some of our own notation and terminology, because there was no “standard” terminology for some of the concepts we tried to clarify. For others, the “standard” terminology is all too often misleading, illogical, obscure, ungrammatical, clumsy, or downright stupid. In these cases, we have not hesitated to introduce our own terminology. Our work led to a lot of written material, for example the books *Finite-Dimensional Spaces, Algebra, Geometry, and Analysis* by myself and *The Basic Language of Mathematics* by J. Schäffer.

I retired from teaching in 1993, but I am still busy with writing, giving lectures, and advising my doctoral student Brian Seguin. This year I was asked to come out of retirement and do Math Studies again with Professor Schäffer. I am looking forward to doing so and also to use the occasion to make some of our material available to the rest of the world by putting it on a special Math Studies website.