On a Variant of the Best Choice Problem with Different Classes of Objects

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This talk considers a variation of the best choice problem in which there are two types of object. There are n objects of each type and an object can be ranked with respect to objects of the same type. At each of 2n moments of time, a decision maker picks an object at random without replacement and must then either reject that object (permanently) and continue searching or accept it and hence stop searching. Hence, the probability that the object picked at a particular time is of type 1 is simply the proportion of the remaining objects which are of type 1. It is assumed that the number n is known to the decision maker. The goal of the decision maker is to accept an object that is the best of its type. Hence, the expected reward under a given strategy can be defined to be the probability of accepting an object which is the best of its type.

Any particular realization of such a problem can be solved using dynamic programming. However, to derive general results regarding the form of such a solution, an approach based on combinatorics is particularly useful. The asymptotic value of the expected reward as $n \to \infty$, 0.5, was derived by Gnedin. However, the derivation of this value relies on the fact that asymptotically, when $n \to \infty$, then the proportion of objects still to be picked that are of type 1 will always be 0.5. In order to describe a full asymptotic solution of such a problem (which is useful in describing the optimal strategy when n is large), it is necessary to consider how the decision maker should act when the process deviates from this asymptotic path.

This talk will give general results regarding the form of the optimal solution for this problem. In addition, we present a full asymptotic solution of this problem. Although this solution cannot be fully described analytically, a numerical solution is given. This solution has been derived in two ways.

This work has been carried out with Michał Morayne, Małgorzata Kuchta and Małgorzata Sułkowska.

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