Multicolor Ramsey Properties of Random Graphs and Hypergraphs

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First we focus on the size-Ramsey number of a path P_n on n vertices. In particular, we show that $5n/2 - 15/2 \leq \hat{r}(P_n) \leq 74n$ for n sufficiently large. This improves the previous lower bound due to Bollobás, and the upper bound due to Letzter. Next we study long monochromatic paths in edge-colored random graph $G(n,p)$ with $pn \to \infty$. Recently, Letzter showed that a.a.s. any 2-edge coloring of $G(n,p)$ yields a monochromatic path of length $(2/3 - o(1))n$, which is optimal. Extending this result, we show that a.a.s. any 3-edge coloring of $G(n,p)$ yields a monochromatic path of length $(1/2 - o(1))n$, which is also optimal. We will also discuss this problem for an arbitrary number of colors. We also consider a related problem and show that for any $r \geq 2$, a.a.s. any r-edge coloring of $G(n,p)$ yields a monochromatic connected subgraph on $(1/(r-1) - o(1))n$ vertices, which is also tight. Finally, we discuss some extensions of the above results for random hypergraphs.

This is a joint work with Paweł Prałat and also with Patrick Bennett, Louis DeBiasio, and Sean English.