Contagious Sets in Dense Graphs

Abstract

We study the activation process in undirected graphs known as bootstrap percolation: A vertex is active either if it belongs to a set of initially activated vertices or if at some point it had at least r active neighbors. The threshold r is identical for all vertices. A contagious set is a subset of vertices whose activation results with the entire graph being active.

In the first part of the talk we prove that G has a contagious set of size 2 if $r = 2$ and G is a Dirac graph, meaning it has minimum degree $n/2$.

In the second part of the talk, we investigate $M(n, k, r)$, the maximum number of edges an n-vertex graph can have without having a contagious set of size k. Noticing that any disconnected graph cannot have a contagious set of size k if all thresholds are k, we find that $M(n, k, k) \geq \binom{n-1}{2}$. In the second part of the talk we then prove that for $n \geq 2k - 2$ this lower bound is in fact tight.

Joint work with Matthias Poloczek and Daniel Reichman