Dynamics for the mean-field random-cluster model

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Abstract
The random-cluster model has been widely studied as a unifying framework for random graphs, spin systems and random spanning trees, but its dynamics have so far largely resisted analysis.

In this work we study a natural non-local Markov chain known as the Chayes-Machta dynamics for the mean-field case of the random-cluster model, and identify a critical regime \((\lambda_s, \lambda_S)\) of the model parameter \(\lambda\) in which the dynamics undergoes an exponential slowdown. Namely, we prove that the mixing time is \(\Theta(\log n)\) if \(\lambda \not\in [\lambda_s, \lambda_S]\), and \(\exp(\Omega(\sqrt{n}))\) when \(\lambda \in (\lambda_s, \lambda_S)\). These results hold for all values of the second model parameter \(q > 1\). In addition, we prove that the local heat-bath dynamics undergoes a similar exponential slowdown in \((\lambda_s, \lambda_S)\).

Joint work with Alistair Sinclair.