Almost-spanning universality in random graphs

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A graph G is said to be $\mathcal{H}(\Delta, n)$-universal if it contains every graph on n vertices with maximum degree at most Δ. A result of Alon, Capalbo, Kohayakawa, Rödl, Ruciński and Szemerédi says that for any $\epsilon > 0$ and any natural number Δ there exists $c > 0$ such that the random graph $\mathcal{G}_{(1+\epsilon)n,p}$ is asymptotically almost surely $\mathcal{H}(\Delta, n)$-universal for $p \geq c(\log n/n)^{1/\Delta}$. This bound has become a benchmark in the field and much subsequent work on embedding spanning or almost-spanning structures of maximum degree Δ in random graphs works down to this threshold. We bypass this bottleneck, showing that there is $\delta_\Delta > 1/\Delta$ such that the same conclusion holds for $p \geq c(\log n/n)^{\delta_\Delta}$.