Defective Crystals

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- "Lattice vector" fields $\boldsymbol{\ell}_1(\cdot), \boldsymbol{\ell}_2(\cdot), \boldsymbol{\ell}_3(\cdot)$, $\boldsymbol{x} \in \Omega \subseteq \mathbb{R}^3$ (Davini; Parry).
  $\{\boldsymbol{d}_a(\boldsymbol{x})\}$ dual fields, $\boldsymbol{d}_a(\boldsymbol{x}) \cdot \boldsymbol{\ell}_b(\boldsymbol{x}) = \delta_{ab}$
  $\oint\limits_{\mathcal{C}} \boldsymbol{d}_a \cdot \boldsymbol{d}x = \int\limits_{\mathcal{C}}\int \nabla \wedge \boldsymbol{d}_a \cdot \boldsymbol{d}x$ Burgers vector
  $\mathcal{S}_{ab} \equiv \frac{\nabla \wedge \boldsymbol{d}_a}{n} \cdot \boldsymbol{d}_b$, $n = \boldsymbol{d}_1 \cdot \boldsymbol{d}_2 \wedge \boldsymbol{d}_3$
  $S_{ab} = \frac{1}{2}\boldsymbol{d}_a \cdot \varepsilon^{bcd}\left[\boldsymbol{\ell}_c, \boldsymbol{\ell}_d\right]$
- Elastic and plastic ('neutral') deformations
- Energy: $w\left(\{\boldsymbol{\ell}_a\}, S, \; \boldsymbol{\ell} \cdot \nabla S, \ldots\right)$. Constitutive assumption: can be truncated, $\rightarrow$ L is a finite dimensional Lie algebra of vector fields
- Discrete structures: discrete subgroups of corresponding Lie group are symmetries of w, as well as create a discrete set of vertices.
- In low dimensional cases transformation groups, proper discrete subgroups be classified.

$H^{1,\infty}$ weak * convergence of minimizers, allowing elastic and neutral plastic deformations.
($S = 0$ case: Cipot/Kinderlehrer; Fonseca/Parry.)