Pattern formation via tight energy bounds (joint work with R. Kohn and S. Muller)

- Introduction to pattern formation in variational problems.
- Emergence of branched microstructures in Kohn-Muller minimization model
- Local energy bounds and self-similarity in the case of 0 relaxed energy.
- Case of nonzero relaxed energy. Global upper and lower energy bounds. Predicted patterns.
- Local energy estimates in the case of nontrivial relaxed energy.
- Extension to the elastic case. Global scaling laws.

Kohn-Muller functional:

$$I_\varepsilon(u) := \int_0^1 \int_0^1 (u_x^2 + \varepsilon |u_{yy}|) dxdy \to \min, u \in A_1 \qquad (1)$$

$$A_1 := \{u \in H^1([0,1]^2), |u_{yy}| = 1, u(0,y) = -y, u(1,y) = y\}.$$

Theorem 1. Let u_ε be a minimizer of $I_\varepsilon[u]$ in A_1. Then

$$\frac{1}{3} + 3^{2/3}\varepsilon^{2/3} \le I(u_\varepsilon) \le \frac{1}{3} + 2^{1/3}3^{2/3}\varepsilon^{2/3}$$

We also consider the elasticity version of this functional. Let

$$J_\varepsilon(u,v) := \int_0^1 \int_0^1 (|u_x + v_y|^2 + v_x^2 + \varepsilon |u_{yy}|) dxdy \to \min \qquad (2)$$

$$A_2 := \{(u,v) : |u_{yy}| = 1, u(0,y) = -y, u(1,y) = y\}.$$

Theorem 2. Let $(u_\varepsilon, v_\varepsilon)$ be a minimizer of $J_\varepsilon[u,v]$ in A_2. Then

$$C_{min}\varepsilon^{4/5} \le J_\varepsilon(u_\varepsilon, v_\varepsilon) \le C_{max}\varepsilon^{4/5}.$$