Activism, Strategic Trading, and Liquidity

Kerry Back
Pierre Collin-Dufresne
Vyacheslav Fos
Tao Li
Alexander Ljungqvist

Overview

- We analyze a Kyle model in which the strategic trader is a potential activist who can affect the value of a stock by exerting costly effort.
- From The Economist: Between 2010 and 2014, half the companies in the S&P 500 index had an activist shareholder and one in seven were the target of an activist campaign.
- We study the relation between market liquidity and activism.
- Main prior paper: Maug (JF, 1998), a single-period Kyle model with binary activism.

Activism Model

- Date T of activism is fixed exogenously. Effort or lack of effort is publicly observed.
- One potential activist. Her blockholding is not publicly observed prior to T.
- The activist's effort is chosen optimally, depending on her blockholding at T.
- The cost of the effort required to produce a share value of v is $C(v) \geq 0$. We take $C(v) = \infty$ if v is infeasible.
- Example (binary): $C(L) = 0$, $C(H) = c > 0$, $C(v) = \infty$ if $v \notin \{L, H\}$.
- Example (quadratic): $C(v) = (v - v_0)^2/(2\psi)$.

Trading Model

- Prior to T, the potential activist can trade profitably on private information about her blockholding (and therefore private information about her intentions).
- After T, she has no private information and therefore cannot profitably trade. So, we assume trading ends at T.
- Trading is continuous during $[0, T]$ (for tractability). Follows Kyle (1985) model:
  - $X_t =$ large trader's position. $X_0 \sim n(\mu_x, \sigma_x^2)$.
  - Noise trades $Z =$ Brownian motion with std dev σ.
  - Risk neutral, competitive market makers.

Value of Shares at T

- Value of x shares to the activist at T is

$$G(x) \stackrel{\text{def}}{=} \sup_v \{vx - C(v)\}.$$

- G is a convex function.
- Assume C is lower semicontinuous and grows more than linearly:

$$\lim_{v \to -\infty, +\infty} \left| \frac{C(v)}{v} \right| = \infty$$

- Then there is an optimal v for the activist. Let $V(x)$ denote an optimum.
- $V(x)$ is a subgradient of G at x. Almost everywhere, $G'(x) = V(x)$ (the marginal value of shares to the activist is the market value – this is the envelope theorem).

Prices before T

- Define $Y_t = (X_t - X_0) + Z_t$. This is the aggregate order process observed by market makers and used by market makers to set prices.
- An equilibrium condition is that the price at each date $t < T$ equal the expected value of $V(X_T)$ conditional on the history of Y until t.
- We look for an equilibrium in which the price at t depends only on Y_t. Let $P(t, y)$ denote the price function.

Optimal Trading

- Assume the potential activist's trades are of order $\mathrm{d}t$ (always true in continuous-time Kyle models).
- So, $\mathrm{d}X = \theta\,\mathrm{d}t$ for some θ.
- Information of the activist at t is X_0 and history of Z until t (can infer Z from prices). Therefore knows $Y_t = (X_t - X_0) + Z_t$.
- Value function of the potential activist is

$$J(t,x,y) = \sup_{\theta} \mathrm{E}\left[G(X_T) - \int_t^T P(u, Y_u)\theta_u \,\mathrm{d}u \,\middle|\, X_t = x, Y_t = y \right]$$

HJB Equation

$$0 = \sup_{\theta} \left\{ -P\theta + J_t + J_x\theta + J_y\theta + \frac{1}{2}\sigma^2 J_{yy} \right\} .$$

Equivalently,

$$-P + J_x + J_y = 0 \,,$$
$$J_t + \frac{1}{2}\sigma^2 J_{yy} = 0 \,.$$

Main Theorem

Define

$$\Lambda = 1 + \sqrt{1 + \frac{\sigma_x^2}{\sigma^2 T}}$$

The pricing rule

$$P(t, y) = \mathrm{E}\left[V(\mu_x + \Lambda y + \Lambda(Z_T - Z_t)\right] \quad (1)$$

and trading strategy

$$\theta_t = \frac{1}{T-t}\left(\frac{X_t - \mu_x - \Lambda Y_t}{\Lambda - 2}\right) \quad (2)$$

constitute an equilibrium.

Main Theorem continued

- The share price post-activism is

$$P(T, Y_T) = V(\mu_x + \Lambda Y_T) = G'(\mu_x + \Lambda Y_T) = G'(X_T)$$

- The distribution of Y given market makers' information is that of a Brownian motion with zero drift and standard deviation σ (the same law as Z).
- The formula for $P(t, Y_t)$ is the expected value of $P(T, Y_T)$ conditional on Y_t and conditional on Y having the same law as Z.
- Market makers view Y as being a Brownian motion with zero drift and the same std dev as Z because the potential activist's trades are 'inconspicuous' (have zero mean) and because a continuous martingale Y with $(\mathrm{d}Y)^2 = \sigma^2\,\mathrm{d}t$ is a Brownian motion with std dev σ

Main Theorem continued

The value function is

$$J(t,x,y) = \frac{\Lambda-1}{\Lambda}\mathsf{E}\left[G\left(\frac{\Lambda(x-Z_T)-\mu_x}{\Lambda-1}\right)\,\middle|\, Z_t = y\right] + \frac{1}{\Lambda}\mathsf{E}\left[G(\mu_x+\Lambda Z_T)\,\middle|\, Z_t = y\right].$$

The equilibrium price evolves as $\mathrm{d}P(t,Y_t) = \lambda(t,Y_t)\,\mathrm{d}Y_t$, where Kyle's lambda is

$$\lambda(t,y) = \frac{\partial P(t,y)}{\partial y}\,. \tag{3}$$

Furthermore, $\lambda(t, Y_t)$ is a martingale on $[0, T-\delta]$ for every $\delta > 0$, relative to the market makers' information set.

Construction of the Value Function

- To construct the value function, we consider the hypothetical (non-equilibrium) strategy of not trading until just before T and then trading until the price equals the marginal value. This value at T is

$$J(T, x, y) \stackrel{\text{def}}{=} G(x) + \sup_{\bar{y}} G(x + \bar{y} - y) - \int_y^{\bar{y}} P(T, u)\,\mathrm{d}u$$

- The value at $t < T$ is the expectation of $J(T, x, Y_T)$ again viewing Y as a Brownian motion with std dev σ.

Lemma

Let ε be a standard normal random variable that is independent of Z. Let b be a nonnegative constant, and set $a = \sigma\sqrt{(2b+1)T}$. Then, the solution Y of the stochastic differential equation

$$\mathrm{d}Y_t = \frac{a\varepsilon - bZ_t - (b+1)Y_t}{T-t}\,\mathrm{d}t + \mathrm{d}Z_t$$

on the time interval $[0, T)$ has the following properties:

- Y is a Brownian motion with zero drift and standard deviation σ on its own filtration on $[0, T]$
- With probability 1,

$$Y_T = \frac{a\varepsilon - bZ_T}{b+1}\,.$$

Remark: The case $b = 0$ is a Brownian bridge.

Comparison to the Standard Kyle Model

- In the standard Kyle model,
  - $\varepsilon = N^{-1}(F(v))$ where N is the standard normal cdf, and F is the cdf of v.
  - $a = \sigma\sqrt{T}$
  - $b = 0$
  - $X_T = X_0 + \sigma\sqrt{T}N^{-1}(F(v)) - Z_T$
- In our model,
  - $\varepsilon = (X_0 - \mu_x)/\sigma_x$
  - $a = \sigma_x/(\Lambda - 2)$
  - $b = 1/(\Lambda - 2)$
  - $X_T = \mu_x + \frac{\Lambda}{\Lambda-1}(X_0 - \mu_x - Z_T) = \mu_x + \Lambda Y_T$

Efficiency

- $P(0,0) = \mathrm{E}[V(T, Y_T)]$ reflects the value per share expected to be created by activism.
- We measure economic efficiency by $P(0,0)$.
- We should subtract the cost of activism, but we assume it is small on a per-share basis (activists cover costs from relatively small percentage shareholdings).

Market Liquidity

- We measure market illiquidty by the expected average Kyle's lambda.
- Because lambda is a martingale,

$$\lambda(0,0) = \mathsf{E} \int_0^T \lambda(t, Y_t)\,\mathrm{d}t$$

- So, we measure illiquidity by $\lambda(0,0)$.

Comparative Statics

- Let $\bar{P}$ denote $P(0,0)$ as a function of model parameters. Let $\bar{\lambda}$ denote $\lambda(0,0)$ as a function of model parameters.
- We are interested in the comparative statics:

$$\frac{\partial \bar{P}}{\partial \sigma}, \qquad \frac{\partial \bar{\lambda}}{\partial \sigma}$$

  and the same with respect to σ_x, μ_x, and parameters of the cost function $C(v)$.
- We have

$$\bar{P} = \mathrm{E}[V(\mu_x + \Lambda Y_T)], \qquad \bar{\lambda} = \Lambda \mathrm{E}[V'(\mu_x + \Lambda Y_T)]$$

Comparative Statics

- Let $\bar{P}$ denote $P(0,0)$ as a function of model parameters. Let $\bar{\lambda}$ denote $\lambda(0,0)$ as a function of model parameters.
- We are interested in the comparative statics:

$$\frac{\partial \bar{P}}{\partial \sigma}, \qquad \frac{\partial \bar{\lambda}}{\partial \sigma}$$

  and the same with respect to σ_x, μ_x, and parameters of the cost function $C(v)$.
- We have

$$\bar{P} = \mathsf{E}[V(\mu_x + \Lambda Y_T)], \qquad \bar{\lambda} = \Lambda \mathsf{E}[V'(\mu_x + \Lambda Y_T)]$$

Comparative Statics: Noise Trading

- $\partial \bar{P}/\partial \sigma \geq 0$ if V is convex and ≤ 0 if V is concave, because an increase in σ is a mean-preserving spread in $\mu_x + \Lambda Y_T$.
- The effect of σ on market liquidity is ambiguous.
- We give an example in which V is affine, other examples in which V is strictly convex, and one example in which V is neither convex nor concave.
- A concave V must be unbounded below, which means that unbounded value destruction is possible.

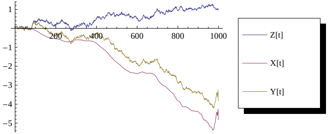
1. Quadratic Cost

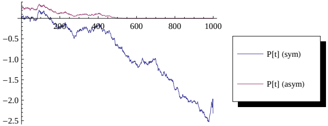
- $C(v) = (v - v_0)^2/(2\psi)$
- Higher ψ means more productive (less cost)
- Activist can destroy value as well as create value
- $V(x) = v_0 + \psi x$ is convex and concave.
- Kyle's lambda is constant over time.
- An increase in σ increases market liquidity but has no effect on efficiency.
- An increase in ψ reduces market liquidity and increases (reduces) efficiency if $\mu_x > 0$ ($\mu_x < 0$)

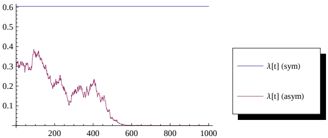
2. Asymmetric Quadratic Cost

- Quadratic for $v > v_0$ and ∞ for $v < v_0$
- $V(x) = v_0 + \psi x^+$
- An increase in σ increases market liquidity and increases efficiency
- Increases in other parameters reduce liquidity and increase efficiency.

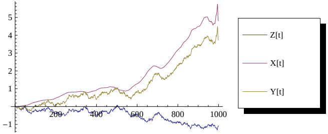
Examples 1 and 2 when Noise Traders Buy

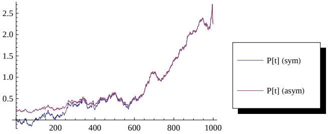


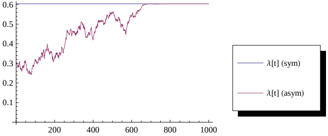




Examples 1 and 2 when Noise Traders Sell





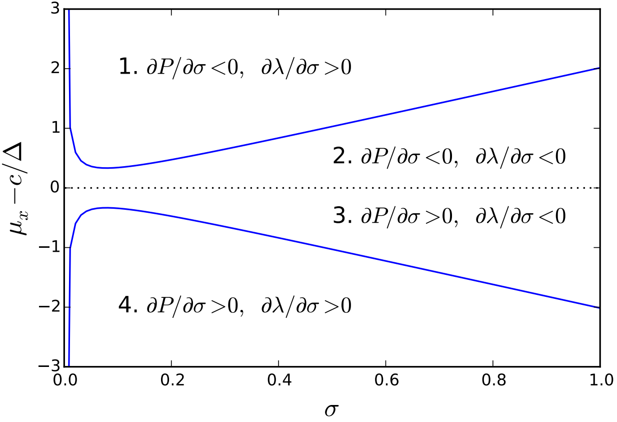


4. Binary

- $C(v_0) = 0,\ C(v_0 + \Delta) = c,\ C(v) = \infty$ otherwise.
- $V(x) = v_0$ if $x\Delta < c$, $V(x) = v_0 + \Delta$ if $x\Delta > c$
- An increase in σ increases efficiency if $\mu_x \Delta > c$ and reduces efficiency if $\mu_x \Delta < c$ (Maug, 1998)
- An increase in σ can either increase or reduce market liquidity.
- Measure productivity by Δ and $\psi \stackrel{\text{def}}{=} \Delta/c$. An increase in either reduces market liquidity and increases efficiency.

Binary Example

Effects of an increase in liquidity trading σ on efficiency $\bar{P}$ and market liquidity $1/\bar{\lambda}$



Conclusion

- Market liquidity and activism are both endogenous. The cross-sectional relation between them depends on the source of cross-sectional variation and on the activism technology.
- Under a natural convexity assumption, an increase in noise trading increases activism. But it may reduce market liquidity.
- An increase in activist productivity generally increases efficiency and reduces market liquidity.