

Putnam $\Sigma.7$

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1 Problems

Putnam 2005/A4. Let H be an $n \times n$ matrix all of whose entries are ± 1 and whose rows are mutually orthogonal. Suppose H has an $a \times b$ submatrix whose entries are all 1. Show that $ab \leq n$.

Putnam 2005/A5. Evaluate $\int_0^1 \frac{\ln(x+1)}{x^2+1} dx$.

Putnam 2005/A6. Let n be given, $n \geq 4$, and suppose that P_1, P_2, \dots, P_n are n randomly, independently and uniformly, chosen points on a circle. Consider the convex n -gon whose vertices are the P_i . What is the probability that at least one of the vertex angles of this polygon is acute?