Putnam Σ.4

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Meet in Wean 5403 at 10:30am

1 Problems

Putnam 2014/A4. Suppose X is a random variable that takes on only nonnegative integer values, with $E[X] = 1$, $E[X^2] = 2$, and $E[X^3] = 5$. (Here $E[y]$ denotes the expectation of the random variable Y.) Determine the smallest possible value of the probability of the event $X = 0$.

Putnam 2014/A5. Let

$$P_n(x) = 1 + 2x + 3x^2 + \cdots + nx^{n-1}.$$

Prove that the polynomials $P_j(x)$ and $P_k(x)$ are relatively prime for all positive integers j and k with $j \neq k$.

Putnam 2014/A6. Let n be a positive integer. What is the largest k for which there exist $n \times n$ matrices $M_1, \ldots, M_k$ and $N_1, \ldots, N_k$ with real entries such that for all i and j, the matrix product $M_i N_j$ has a zero entry somewhere on its diagonal if and only if $i \neq j$?