

Putnam E.12

Po-Shen Loh

18 November 2025

1 Problems

Putnam 2007/B1. Let f be a nonconstant polynomial with positive integer coefficients. Prove that if n is a positive integer, then $f(n)$ divides $f(f(n) + 1)$ if and only if $n = 1$.

Putnam 2007/B2. Suppose that $f : [0, 1] \rightarrow \mathbb{R}$ has a continuous derivative and that $\int_0^1 f(x) dx = 0$. Prove that for every $\alpha \in (0, 1)$,

$$\left| \int_0^\alpha f(x) dx \right| \leq \frac{1}{8} \max_{0 \leq x \leq 1} |f'(x)|.$$

Putnam 2007/B3. Let $x_0 = 1$ and for $n \geq 0$, let $x_{n+1} = 3x_n + \lfloor x_n \sqrt{5} \rfloor$. In particular, $x_1 = 5$, $x_2 = 26$, $x_3 = 136$, $x_4 = 712$. Find a closed-form expression for x_{2007} . ($\lfloor a \rfloor$ means the largest integer $\leq a$.)