

# Putnam E.9

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## 1 Problems

**Putnam 2006/B1.** Show that the curve  $x^3 + 3xy + y^3 = 1$  contains only one set of three distinct points  $A$ ,  $B$ , and  $C$ , which are vertices of an equilateral triangle, and find its area.

**Putnam 2006/B2.** Prove that for every set  $X = \{x_1, \dots, x_n\}$  of real numbers, there exists a non-empty subset  $S$  of  $X$  and an integer  $m$  such that

$$\left| m + \sum_{s \in S} s \right| \leq \frac{1}{n+1}.$$

**Putnam 2006/B3.** Let  $S$  be a finite set of points in the plane. A linear partition of  $S$  is an unordered pair  $\{A, B\}$  of subsets of  $S$  such that  $A \cup B = S$ ,  $A \cap B = \emptyset$ , and  $A$  and  $B$  lie on opposite sides of some straight line disjoint from  $S$  ( $A$  or  $B$  may be empty). Let  $L_S$  be the number of linear partitions of  $S$ . For each positive integer  $n$ , find the maximum of  $L_S$  over all sets  $S$  of  $n$  points.