

Putnam E.2

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1 Problems

Putnam 2005/B1. Find a nonzero polynomial $P(x, y)$ such that $P(\lfloor a \rfloor, \lfloor 2a \rfloor) = 0$ for all real numbers a .
(Note: $\lfloor \nu \rfloor$ is the greatest integer less than or equal to ν .)

Putnam 2005/B2. Find all positive integers n, k_1, \dots, k_n such that $k_1 + \dots + k_n = 5n - 4$ and

$$\frac{1}{k_1} + \dots + \frac{1}{k_n} = 1.$$

Putnam 2005/B3. Find all differentiable functions $f : (0, \infty) \rightarrow (0, \infty)$ for which there is a positive real number a such that

$$f' \left(\frac{a}{x} \right) = \frac{x}{f(x)}$$

for all $x > 0$.