Putnam Σ.10

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3 November 2024

1 Problems

Prof. Loh will not be in town to lead the discussion and the dinner, but if you wish to sit together in Wean 5403 and discuss on your own at 6pm, you are very welcome to!

Putnam 1997/B4. Let $a_{m,n}$ denote the coefficient of x^n in the expansion of $(1+x+x^2)^m$. Prove that for all integers $k \geq 0$,

$$0 \leq \sum_{i=0}^{\lfloor \frac{2k}{3} \rfloor} (-1)^i a_{k-i,i} \leq 1.$$

Putnam 1997/B5. Prove that for $n \geq 2$,

$$\underbrace{2^{2^{\cdots^2}}}_{n \text{ terms}} \equiv \underbrace{2^{2^{\cdots^2}}}_{n-1 \text{ terms}} \pmod{n}.$$

Putnam 1997/B6. The dissection of the 3–4–5 triangle into four congruent right triangles similar to the original has diameter 5/2. (The diameter of a dissection is the least upper bound of the distances between pairs of points belonging to the same part.) Find the least diameter of a dissection of this triangle into four parts.