

# Putnam $\Sigma.9$

Po-Shen Loh

27 October 2024

## 1 Problems

**Putnam 1997/A4.** Let  $G$  be a group with identity  $e$  and  $\phi : G \rightarrow G$  a function such that

$$\phi(g_1)\phi(g_2)\phi(g_3) = \phi(h_1)\phi(h_2)\phi(h_3)$$

whenever  $g_1g_2g_3 = e = h_1h_2h_3$ . Prove that there exists an element  $a \in G$  such that  $\psi(x) = a\phi(x)$  is a homomorphism (i.e.  $\psi(xy) = \psi(x)\psi(y)$  for all  $x, y \in G$ ).

**Putnam 1997/A5.** Let  $N_n$  denote the number of ordered  $n$ -tuples of positive integers  $(a_1, a_2, \dots, a_n)$  such that  $1/a_1 + 1/a_2 + \dots + 1/a_n = 1$ . Determine whether  $N_{10}$  is even or odd.

**Putnam 1997/A6.** For a positive integer  $n$  and any real number  $c$ , define  $x_k$  recursively by  $x_0 = 0$ ,  $x_1 = 1$ , and for  $k \geq 0$ ,

$$x_{k+2} = \frac{cx_{k+1} - (n-k)x_k}{k+1}.$$

Fix  $n$  and then take  $c$  to be the largest value for which  $x_{n+1} = 0$ . Find  $x_k$  in terms of  $n$  and  $k$ ,  $1 \leq k \leq n$ .