Control Volume Approximation of Degenerate Two Phase Porous Flows

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Corrigendum

- The expression for $\Gamma(\mathbf{s})$ at the beginning of Section 2.2 should read[1]:

  The pressure–saturation relation at each point $x \in \Omega$ can be written as $\mathbf{p} = \partial(I_L + \Gamma)(\mathbf{s})$, where $I_L, \Gamma : \mathbb{R}^2 \to \mathbb{R} \cup \{\infty\}$ are the convex functions $\Gamma(\mathbf{s}) = \gamma(s_1)$ and

$$I_L(\mathbf{s}) = \begin{cases} 0 & s_1 + s_2 = 1 - s_0(x) \\ \infty & \text{otherwise} \end{cases} \quad \text{with } \gamma(s_1) \equiv \begin{cases} \tilde{\gamma}(2s_1 - (1 - s_0)) & 0 \le s_1 \le 1 - s_0(x) \\ \infty & \text{otherwise,} \end{cases}$$

- The subgradient of of $I_L + \Gamma$ in the following paragraph should read:

$$\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} \in p \begin{pmatrix} 1 \\ 1 \end{pmatrix} + (1/2)\, \partial\gamma(s_1) \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

- Equation (2.3) and the chain rule preceding it should read:

  Under mild regularity assumptions, the chain rule for sub–gradients [35] states that

$$\mathbf{p} \in \partial(I_L + \Gamma)(\mathbf{s}) \qquad \Rightarrow \qquad (\mathbf{s}_t, \mathbf{p}) = \frac{d}{dt}(I_L + \Gamma)(\mathbf{s}),$$

  so the natural a–priori estimate for equations (2.1)–(2.2) with no–flux boundary data is

$$\int_\Omega \gamma(s_1(t)) + \int_0^t \int_\Omega \sum_{\pi=1}^{2} |\nabla p_\pi|^2_{K_\pi} = \int_\Omega \gamma(s_1(0)) + \int_0^t \int_\Omega \sum_{\pi=1}^{2} (\mathbf{b}_\pi, \nabla p_\pi)_{K_\pi}. \tag{2.3}$$

- The text following equation (2.5) should read:

  Convexity of γ guarantees

$$\left((\gamma^*)'(p_1^n - p_2^n) - s_1^{n-1}\right)(p_1^n - p_2^n) = \left(s_1^n - s_1^{n-1}\right)\gamma'(s_1^n) \ge \gamma(s_1^n) - \gamma(s_1^{n-1}),$$

  from which the discrete analog of (2.3) for solutions of the implicit Euler scheme (2.4) follows,

$$\int_\Omega \gamma(s_1^n) + \tau \sum_{m=1}^{n} \int_\Omega \sum_{\pi=1}^{2} |\nabla p_\pi^m|^2_{K_\pi^{m-1}} \le \int_\Omega \gamma(s_1^0) + \tau \sum_{m=1}^{n} \int_\Omega \sum_{\pi=1}^{2} (\nabla p_\pi^m, \mathbf{b}_\pi^{m-1})_{K_\pi^{m-1}}.$$

[1] The juxtaposition of γ and $\tilde{\gamma}$ is made since in the engineering texts the saturation pressure is typically prescribed as a function of s_1.