Test 3 August 11

Name:

1. Solve each of the following differential equations

Livear Aiff. eg.
$$y' + y = x^{3/2}e^{-x}$$
 $y(0) = -2$

$$I(x) = e^{\int Idx} = e^{x} + \lambda$$

$$e^{x}y' + e^{x}y = x^{3/2}$$

$$(e^{x}y)' = \int x^{3/2}$$

$$e^{x}y = \frac{2}{5}x^{5/2} + \lambda$$

$$y = \frac{2}{5}x^{5/2}e^{-x} + \lambda$$

$$-2 = 0 + C$$

$$C = -2$$

$$Y = \frac{2}{5} x^{5/2} e^{-x} - 2e^{-x}$$

$$y' = \frac{x^2 + 1}{1 + \sin y}$$
Separable +3
$$\int 1 + \sin y \, dy = \int x^2 + 1 \, dx + 4$$

$$y - \cos y = \frac{x^3}{3} + x + C + \frac{1}{3}$$

(c)
$$y'' - y' - 6y = 0$$
Aux. Eqn: $r^2 - r - 6 = 0 + 3$

$$(r-3)(r+2) = 0$$

$$r = +3 \text{ or } -2 + 3$$

$$y(x) = c_1 e^{3x} + c_2 e^{-2x} + 4$$

(d)
$$y'' - 4y' + 5y = 0$$

Aux. Eqn.
$$r^2 - 4r + 5 = 0 + 3$$

$$r = \frac{4 \pm (16 - 41.5)}{2} = 2 \pm \frac{54}{2} = 2 \pm 1$$

$$Y = e^{2x} (c_1 (osx + c_2 sin x) + 5$$

$$y'' - 4y' + 4y = \sin(4x)$$

Find ye !

Aux rgn:
$$r^2 - 4r + 4 = 0$$

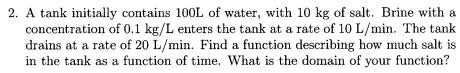
 $(r-2)^2 = 0$

$$y_c = c_1 e^{2x} + c_2 x e^{2x} + 3$$

Find Yp:

$$-\frac{100}{3}B = 1 \rightarrow B = -\frac{3}{100}$$

$$A = -\frac{4}{3} \cdot \frac{-3}{100} = \frac{1}{25}$$



$$\frac{dy}{dt} = rate in - rate out$$
= 0.1.10 - $\left(\frac{y}{100-10t}\right)$ 20 + 2

$$Y' + (\frac{20}{100-10t})Y = 1$$

$$T(t) = e^{\int \frac{20}{100-10t}dt} = e^{2\ln|100-10t|} = (100-10t)^{-2} + 2$$

$$\int ((100-10t)^{-2}y)' = \int (100-10t)^{-2} d^{4}$$

$$(100-10t)^{-2}y = -(100-10t)^{-1} + C + 2$$

$$Y = -(100 - 10t) + ((100 - 10t)^{*2}$$

$$\gamma(0) = 10$$
: $10 = 0 - 10 + C(100 - 0)^2$

$$20 = ((100)^2 + 2)$$

$$(= \frac{20}{100^2} = \frac{2}{1000}$$

$$\gamma = t - 10 + \frac{2(100 - 104)^2}{1000} + 1$$

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- 3. Suppose pure sodium (Na) is pumped into a tank at a constant rate 5 kg/min. As it is pumped in, it reacts with chlorine (Cl) and it is converted into salt (NaCl) at a rate proportional to the concentration at the time (assume there is an infinite supply of Cl), with a proportionality constant of 2. Let y(t) = the amount of pure sodium in the tank at time t
 - (a) Explain why y is a solution to the following differential equation:

$$y' = 5 - 2y$$

(b) Solve this differential equation, and explain what happens as time goes on.

b)
$$y' + 2y = 5$$

$$I(t) = e^{\int 2dt} = e^{2t}$$

$$(e^{2t}y)' = (5e^{2t} = \frac{5}{2}e^{2t} + C + 3)$$

$$y = \frac{5}{2} + Ce^{-2t} + 3$$

- 4. A spring has a weight with a mass of 5 attached to the end of it. If x is the distance the weight is away from equilibrium, the resisting force of the spring is -5x.
 - (a) Give an equation describing the position of the mass as a function of time.
 - (b) You intend to damp the vibrations with a viscous fluid with damping force -cx'. Descibe what consequences choosing different values of c will have.

a)
$$Ma = f$$

$$5x'' = -5x + 2$$

$$5x'' + 5x = 0$$

$$x'' + x = D$$

$$Aux cgn: r^2 + 1 = 0$$

$$r = \pm \lambda$$

$$X = c_1 cost + c_2 sint + 3$$

Aux:
$$5r^2 + \epsilon r + 5 = 0$$

$$r = \frac{-c \pm \sqrt{c^2 - 100}}{10} + 2$$