Test 3

August 11

Name:

1. Solve each of the following differential equations

   (a)
$$y' + y = x^{3/2}e^{-x} \quad y(0) = -2$$

(b)

$$y' = \frac{x^2 + 1}{1 + \sin y}$$

(c)

$$y'' - y' - 6y = 0$$

(d)

$$y'' - 4y' + 5y = 0$$

(e)

$$y'' - 4y' + 4y = \sin(4x)$$

2. A tank initially contains 100L of water, with 10 kg of salt. Brine with a concentration of 0.1 kg/L enters the tank at a rate of 10 L/min. The tank drains at a rate of 20 L/min. Find a function describing how much salt is in the tank as a function of time. What is the domain of your function?

3. Suppose pure sodium (Na) is pumped into a tank at a constant rate 5 kg/min. As it is pumped in, it reacts with chlorine (Cl) and it is converted into salt (NaCl) at a rate proportional to the concentration at the time (assume there is an infinite supply of Cl), with a proportionality constant of 2. Let $y(t) =$ the amount of pure sodium in the tank at time t

   (a) Explain why y is a solution to the following differential equation:

   $$y' = 5 - 2y$$

   (b) Solve this differential equation, and explain what happens as time goes on.

4. A spring has a weight with a mass of 5 attached to the end of it. If x is the distance the weight is away from equilibrium, the resisting force of the spring is $-5x$.

   (a) Give an equation desctibing the position of the mass as a function of time.

   (b) You intend to damp the vibrations with a viscous fluid with damping force $-cx'$. Descibe what consequences choosing different values of c will have.