Test 1
July 14

Name:

1. Evaluate each of the following integrals. If any of them are divergent, show why.

   (a)
$$\int \csc^6 x \; dx$$

(b)

$$\int \frac{\sqrt{x^2 - 9}}{x^3}\, dx$$

(c)

$$\int_0^3 \frac{dx}{x^2 - x - 2}$$

(d)

$$\int_2^6 \frac{y}{\sqrt{y-2}}\, dy$$

2. Find the arc length of the curve $y = \ln(\sec x)$, where $0 \leq x \leq \pi/4$

3. Find the area of the surface obtained by rotating the curve $y = 1 - x^2, 0 \leq x \leq 1$ about the x-axis.

4. For which values of p does the following integral converge? Evaluate the integral for those values.

$$\int_1^{\infty} \frac{\ln x}{x^p} \, dx$$

5. (a) Write the approximations L_4, M_4, and S_4 for

$$\int_0^1 e^{-x^2/2}\, dx$$

Do not evaluate.

(b) Is L_4 an over-estimate or under-estimate for the integral? Explain.

(c) How large must n be to guarantee M_n is within 10^{-5} of the true value?