Quiz 6 Solutions

July 26

Name:

1. Find the interval of convergence for the following series:

$$\sum_{n=1}^{\infty} \frac{(x+4)^n}{\sqrt{n}}$$

Use the Ratio Test:

$$\begin{aligned}
\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| &= \lim_{n\to\infty} \left|\frac{(x+4)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{(x+4)^n}\right| \\
&= \lim_{n\to\infty} \left|(x+4)\sqrt{\frac{n}{n+1}}\right| \\
&= |x+4|
\end{aligned}$$

Thus the series converges if $|x+4| < 1$, i.e. if $-5 < x < -3$. We need to test the endpoints of this interval.
If $x = -5$, the series becomes

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

which converges by the Alternating Series Test.
If $x = -3$, the series becomes

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$$

which diverges since $1/2 < 1$.

This the interval of convergence is $[-5, -3)$.

2. Find a power series representation for the function $f(x) = 5/(3-x)$, and find the interval of convergence for this power series.

$$f(x) = 5/(3-x) = \frac{5}{3} \cdot \frac{1}{1-(x/3)} = \frac{5}{3} \sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n$$

The power series is a geometric series with ratio $x/3$, so it converges if $|x/3| < 1$, i.e. if $-3 < x < 3$.