Quiz 5

July 21

Name:

Test each of the following series for convergence. Explain why each is convergent or divergent, and find the exact sum if possible.

1.
$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

Since $1/n \ln n$ is continuous, decreasing, and positive, we can use the integral test:

$$\begin{aligned}
\int_2^{\infty} \frac{dx}{x \ln x} &= \lim_{t \to \infty} \int_2^t \frac{dx}{x \ln x} \\
&= \lim_{t \to \infty} [\ln(\ln x)]_2^t \\
&= \infty
\end{aligned}$$

The integral diverges, and thus so does the series.

2.

$$\sum_{n=1}^{\infty} \frac{1+n+n^2}{\sqrt{1+n^2+n^6}}$$

The dominant term on the top is n^2, and the dominant term on the bottom is n^6, so the bottom behaves like n^3. Thus the series should behave like $\sum \frac{1}{n}$, and thus diverge. We use the limit comparison test to prove this:

$$\lim_{n\to\infty} \frac{\frac{1+n+n^2}{\sqrt{1+n^2+n^6}}}{1/n} = \lim_{n\to\infty} \frac{n+n^2+n^3}{\sqrt{1+n^2+n^6}} = \lim_{n\to\infty} \frac{1/n^2+1/n+1}{\sqrt{1/n^6+1/n^4+1}} = 1$$

Since $0 < 1 < \infty$, the limit comparison test tells us that our series diverges.

3.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(2n+1)}{5n-3}$$

The series diverges by the test for divergence, since

$$\lim_{n\to\infty} \frac{(-1)^{n-1}(2n+1)}{5n-3} \neq 0$$

4.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1} e^{1/n}}{n}$$

This is an alternating series, and converges by the alternating series test, since the terms $\frac{e^{1/n}}{n}$ are decreasing and approach zero as n gets large.

5.

$$\sum_{n=1}^{\infty} 3 \cdot \frac{2^{2n}}{5^n}$$

This is a geometric series with ratio 4/5, and thus it converges. The first term is 12/5, and so the sum is

$$\frac{12/5}{1 - 4/5} = 12$$

6.
$$\sum_{n=1}^{\infty} \frac{e^n - 10n}{4^n + n^2 + 8n}$$

Since
$$\frac{e^n - 10n}{4^n + n^2 + 8n} \le \frac{e^n}{4^n}$$
and $\sum (e/4)^n$ is a convergent geometric series ($e/4 < 1$), the series converges by the comparison test.