Quiz 3

July 7

Name:

1. Evaluate:

$$\int p^5 \ln p \; dp$$

Use parts, with $u = \ln p, du = 1/p \; dp, dv = p^5 \; dp, v = p^6/6$, to get

$$\begin{aligned}\int p^5 \ln p \; dp &= \frac{p^6}{6} \ln p - \int \frac{p^5}{6} + C \\ &= \frac{p^6}{6} \ln p - \frac{p^6}{36} + C\end{aligned}$$

2. Evaluate:

$$\int_0^{1/2} \frac{x}{\sqrt{1-x^2}}\ dx$$

Use trig. substitution, with $x = \sin\theta, dx = \cos\theta\ d\theta$, to get

$$\begin{aligned}
\int_0^{1/2} \frac{x}{\sqrt{1-x^2}}\ dx &= \int_0^{\pi/6} \frac{\sin\theta\cos\theta}{\sqrt{1-\sin^2\theta}}\ d\theta \\
&= \int_0^{\pi/6} \frac{\sin\theta\cos\theta}{\cos\theta}\ d\theta \\
&= \int_0^{\pi/6} \sin\theta\ d\theta \\
&= [-\cos\theta]_0^{\pi/6} \\
&= 1 - \frac{\sqrt{3}}{2}
\end{aligned}$$

3. Evaluate:

$$\int \frac{1}{3x^2 + 11x + 6}\, dx$$

Use a partial fraction decomposition:

$$\frac{1}{3x^2 + 11x + 6} = \frac{1}{(3x+2)(x+3)} = \frac{A}{3x+2} + \frac{B}{x+3}$$

We get the equation

$$1 = A(x+3) + B(3x+2)$$

Plugging in -3 for x, we find $B = -1/7$, and thus $A = 3/7$.
Thus,

$$\begin{aligned} \int \frac{1}{3x^2 + 11x + 6}\, dx &= \int \frac{3/7}{3x+2} + \frac{-1/7}{x+3}\, dx \\ &= \frac{1}{7} \ln|3x+2| - \frac{1}{7} \ln|x+3| + C \end{aligned}$$

4. Evaluate:

$$\int \frac{dx}{(5-4x-x^2)^{5/2}}$$

$$\begin{aligned}
\int \frac{dx}{(5-4x-x^2)^{5/2}} &= \int \frac{dx}{\sqrt{9-(x+2)^2}^5} \quad \text{complete the square} \\
&= \int \frac{du}{\sqrt{9-u^2}^5} \quad \text{let } u = x+2 \\
&= \int \frac{3\cos y \; dy}{\sqrt{9(1-sin^2 y)}^5} \quad \text{let } u = 3\sin y \\
&= \int \frac{3\cos y \; dy}{(3\cos y)^5} \\
&= \frac{1}{3^4} \int \sec^4 y \; dy \\
&= \frac{1}{3^4} \int (\tan^2 y + 1)\sec^2 y \; dy \\
&= \frac{1}{3^4} \int v^2 + 1 \; dv \quad \text{let } v = \tan y \\
&= \frac{1}{3^4} \left[\frac{v^3}{3} + v \right] + C \\
&= \frac{1}{3^4} \left[\frac{\tan^3 y}{3} + \tan y \right] + C \\
&= \frac{1}{3^4} \left[\frac{u^3}{3\sqrt{9-u^2}^3} + \frac{u}{9-u^2} \right] + C \\
&= \frac{1}{3^4} \left[\frac{(x+2)^3}{3\sqrt{9-(x+2)^2}^3} + \frac{x+2}{9-(x+2)^2} \right] + C
\end{aligned}$$