

Quiz 2 Solutions
July 5

Name:

1. Evaluate:

$$\int x^2 \sin(\pi x) \, dx$$

Use integration by parts: Let $u = x^2$, $dv = \sin(\pi x) \, dx$. Then $du = 2x \, dx$, and $v = -\cos(\pi x)/\pi$. Thus we obtain:

$$\int x^2 \sin(\pi x) dx = -\frac{1}{\pi} x^2 \cos(\pi x) \, dx + \int \frac{2}{\pi} x \cos(\pi x) \, dx + C$$

Applying parts one more time with $u = x$ ($du = dx$), and $dv = \cos(\pi x) \, dx$ ($v = \sin(\pi x)/\pi$), we obtain

$$\begin{aligned} -\frac{1}{\pi} x^2 \cos(\pi x) + \int \frac{2}{\pi} x \cos(\pi x) \, dx + C &= -\frac{1}{\pi} x^2 \cos(\pi x) + \frac{2}{\pi} \left[\frac{1}{\pi} x \sin(\pi x) - \int \frac{1}{\pi} \sin(\pi x) \, dx \right] + C \\ &= -\frac{1}{\pi} x^2 \cos(\pi x) + \frac{2}{\pi^2} x \sin(\pi x) + \frac{2}{\pi^3} \cos(\pi x) + C \end{aligned}$$