Quiz 1 Solutions

June 28

Name:

1. Let

$$K = \int_4^{12} \frac{1}{x} dx$$

Find an approximation for K using right endpoints and $\Delta x = .5$. Express it using sigma notation. Is your approximation an under-estimate or over-estimate? Explain why.

Since $\Delta x = .5$, there will be 16 rectangles, so we will let i go from 1 to 16. The right endpoints of the rectangles come at $x = 4.5, 5, 5.5, \ldots, 12$, which gives the formula $x = 4 + .5i$ for these values of i. We get the following approximation:

$$K \approx \sum_{i=1}^{16} .5 \cdot \frac{1}{4 + .5i}$$

This is an underestimate, since $1/x$ is a decreasing positive function in this range.

Other accptable answers are

$$K \approx \sum_{i=0}^{15} .5 \cdot \frac{1}{4.5 + .5i}$$

or

$$K \approx \sum_{i=1}^{16} .5 \cdot \frac{1}{4.5 + .5(i-1)}$$

or

$$K \approx \sum_{i=0}^{15} .5 \cdot \frac{1}{4 + .5(i+1)}$$

etc...

2. Give a single expression for the following sum:

$$\sum_{i=1}^{13} 5 \cdot 3^{i+2}$$

$$\begin{aligned}
\sum_{i=1}^{13} 5 \cdot 3^{i+2} &= 5 \cdot 3^3 + 5 \cdot 3^4 + \cdots + 5 \cdot 3^{15} \\
&= 5 \cdot 3^3 (1 + 3 + 3^2 + \cdots + 3^{12}) \\
&= 5 \cdot 3^3 \cdot \frac{1 - 3^{13}}{1 - 3}
\end{aligned}$$