6. $\sum_{n=1}^{\infty} \frac{1}{n^4}$ is a convergent p-series ($p = 4 > 1$), so $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$ is absolutely convergent.

8. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ diverges by the Limit Comparison Test with the harmonic series:
$\lim_{n\to\infty} \frac{n/(n^2+1)}{1/n} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$. But $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+1}$ converges by the Alternating Series Test:
$\left\{\frac{n}{n^2+1}\right\}$ has positive terms, is decreasing since $\left(\frac{x}{x^2+1}\right)' = \frac{1-x^2}{(x^2+1)^2} \le 0$ for $x \ge 1$, and
$\lim_{n\to\infty} \frac{n}{n^2+1} = 0$. Thus, $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+1}$ is conditionally convergent.

[illegible]. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)!/e^{n+1}}{n!/e^n}\right| = \frac{1}{e} \lim_{n\to\infty} (n+1) = \infty$, so the series $\sum_{n=1}^{\infty} e^{-n} n!$ diverges by the Ratio Test.

18. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(n+1)!/(n+1)^{n+1}}{n!/n^n} = \lim_{n\to\infty} \frac{n^n}{(n+1)^n} = \lim_{n\to\infty} \frac{1}{(1+1/n)^n} = \frac{1}{e} < 1$, so the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converges absolutely by the Ratio Test.

[illegible]. Use the Ratio Test with the series $\frac{2}{5} + \frac{2\cdot 6}{5\cdot 8} + \frac{2\cdot 6\cdot 10}{5\cdot 8\cdot 11} + \frac{2\cdot 6\cdot 10\cdot 14}{5\cdot 8\cdot 11\cdot 14} + \cdots = \sum_{n=1}^{\infty} \frac{2\cdot 6\cdot 10\cdot 14\cdot\cdots\cdot(4n-2)}{5\cdot 8\cdot 11\cdot 14\cdot\cdots\cdot(3n+2)}$.

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{2\cdot 6\cdot 10\cdot\cdots\cdot(4n-2)[4(n+1)-2]}{5\cdot 8\cdot 11\cdot\cdots\cdot(3n+2)[3(n+1)+2]} \cdot \frac{5\cdot 8\cdot 11\cdot\cdots\cdot(3n+2)}{2\cdot 6\cdot 10\cdot\cdots\cdot(4n-2)}\right|$$
$$= \lim_{n\to\infty} \frac{4n+2}{3n+5} = \frac{4}{3} > 1,$$

so the given series is divergent.

33. (a) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n}\right| = \lim_{n\to\infty} \left|\frac{x}{n+1}\right| = |x| \lim_{n\to\infty} \frac{1}{n+1} = |x| \cdot 0 = 0 < 1$, so by the Ratio Test the series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges for all x.

(b) Since the series of part (a) always converges, we must have $\lim_{n\to\infty} \frac{x^n}{n!} = 0$ by Theorem 11.2.6.

[illegible]. Let $f(x) = x^2 e^{-x^3}$. Then f is continuous and positive on $[1, \infty)$, and $f'(x) = \frac{x(2-3x^3)}{e^{x^3}} < 0$ for $x \ge 1$, so f is decreasing on $[1, \infty)$ as well, and we can apply the Integral Test. $\int_1^\infty x^2 e^{-x^3}\, dx = \lim_{t\to\infty} \left[-\frac{1}{3} e^{-x^3}\right]_1^t = \frac{1}{3e}$, so the integral converges, and hence, the series converges.

[illegible]. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \left(\frac{n^2+2n+2}{5^{n+1}} \cdot \frac{5^n}{n^2+1}\right) = \lim_{n\to\infty} \left(\frac{1+2/n+2/n^2}{1+1/n^2} \cdot \frac{1}{5}\right) = \frac{1}{5} < 1$, so $\sum_{n=1}^{\infty} \frac{n^2+1}{5^n}$ converges by the Ratio Test.

6. $a_n = \sqrt{n} x^n$, so we need $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{\sqrt{n+1}\,|x|^{n+1}}{\sqrt{n}\,|x|^n} = \lim_{n\to\infty} \sqrt{1+\frac{1}{n}}\,|x| = |x| < 1$ for convergence (by the Ratio Test), so $R = 1$. When $x = \pm 1$, $\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \sqrt{n} = \infty$, so the series diverges by the Test for Divergence. Thus, $I = (-1, 1)$.

14. $a_n = (-1)^n \dfrac{x^{2n}}{(2n)!}$, so $\lim\limits_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty} \dfrac{|x|^{2n+2}}{(2n+2)!} \cdot \dfrac{(2n)!}{|x|^{2n}} = \lim\limits_{n\to\infty} \dfrac{|x|^2}{(2n+1)(2n+2)} = 0$. Thus, by the Ratio Test, the series converges for *all* real x and we have $R = \infty$ and $I = (-\infty, \infty)$.

20. $\lim\limits_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty} \left|\dfrac{(3x-2)^{n+1}}{(n+1)\,3^{n+1}} \cdot \dfrac{n3^n}{(3x-2)^n}\right| = \lim\limits_{n\to\infty} \left(\dfrac{|3x-2|}{3} \cdot \dfrac{1}{1+1/n}\right) = \dfrac{|3x-2|}{3} = \left|x - \frac{2}{3}\right|$, so by the Ratio Test, the series converges when $\left|x - \frac{2}{3}\right| < 1 \quad \Leftrightarrow \quad -\frac{1}{3} < x < \frac{5}{3}$. $R = 1$. When $x = -\frac{1}{3}$, the series is $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{n}$, the convergent alternating harmonic series. When $x = \frac{5}{3}$, the series becomes the divergent harmonic series. Thus, $I = \left[-\frac{1}{3}, \frac{5}{3}\right)$.

24. $a_n = \dfrac{n^2 x^n}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot (2n)} = \dfrac{n^2 x^n}{2^n n!} = \dfrac{n x^n}{2^n (n-1)!}$, so

$\lim\limits_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty} \dfrac{(n+1)\,|x|^{n+1}}{2^{n+1} n!} \cdot \dfrac{2^n (n-1)!}{n\,|x|^n} = \lim\limits_{n\to\infty} \dfrac{n+1}{n^2} \dfrac{|x|}{2} = 0$. Thus, by the Ratio Test, the series converges for *all* real x and we have $R = \infty$ and $I = (-\infty, \infty)$.

26. If $a_n = \dfrac{(-1)^n (2x+3)^n}{n \ln n}$, then we need $\lim\limits_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = |2x+3| \lim\limits_{n\to\infty} \dfrac{n \ln n}{(n+1)\ln(n+1)} = |2x+3| < 1$ for convergence, so $-2 < x < -1$ and $R = \frac{1}{2}$. When $x = -2$, $\sum\limits_{n=2}^{\infty} a_n = \sum\limits_{n=2}^{\infty} \dfrac{1}{n \ln n}$, which diverges (Integral Test), and when $x = -1$, $\sum\limits_{n=2}^{\infty} a_n = \sum\limits_{n=2}^{\infty} \dfrac{(-1)^n}{n \ln n}$, which converges (Alternating Series Test), so $I = (-2, -1]$.