[illegible] $\{-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \dots\}$. The numerator of the nth term is n and its denominator is $(n+1)^2$. Including the alternating signs, we get $a_n = (-1)^n \dfrac{n}{(n+1)^2}$.

18. $a_n = \dfrac{\sqrt{n}}{1+\sqrt{n}} = \dfrac{1}{1/\sqrt{n}+1}$, so $a_n \to \dfrac{1}{0+1} = 1$ as $n \to \infty$. Converges

28. $a_n = \dfrac{\ln n}{\ln 2n} = \dfrac{\ln n}{\ln 2 + \ln n} = \dfrac{1}{\frac{\ln 2}{\ln n} + 1} \to \dfrac{1}{0+1} \to 1$ as $n \to \infty$. Converges

[illegible] (a) Let $\lim_{n\to\infty} a_n = L$. By Definition 1, this means that for every $\varepsilon > 0$ there is an integer N such that $|a_n - L| < \varepsilon$ whenever $n > N$. Thus, $|a_{n+1} - L| < \varepsilon$ whenever $n + 1 > N \iff n > N - 1$. It follows that $\lim_{n\to\infty} a_{n+1} = L$ and so $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$.

(b) If $L = \lim_{n\to\infty} a_n$ then $\lim_{n\to\infty} a_{n+1} = L$ also, so L must satisfy $L = 1/(1+L) \Rightarrow L^2 + L - 1 = 0 \Rightarrow$ $L = \frac{-1+\sqrt{5}}{2}$ (since L has to be non-negative if it exists).

60. $a_n = n + \dfrac{1}{n}$ defines an increasing sequence since the function $g(x) = x + \dfrac{1}{x}$ is increasing for $x > 1$. $[g'(x) = 1 - 1/x^2 > 0$ for $x > 1.]$ The sequence is unbounded since $a_n \to \infty$ as $n \to \infty$. (It is, however, bounded below by $a_1 = 2$.)

12. $\frac{1}{8} - \frac{1}{4} + \frac{1}{2} - 1 + \cdots$ is a geometric series with $r = -2$. Since $|r| = 2 > 1$, the series diverges.

16. $\displaystyle\sum_{n=1}^{\infty} \frac{(-6)^{n-1}}{5^{n-1}}$ is a geometric series with $a = 1$ and $r = -\frac{6}{5}$. The series diverges since $|r| = \frac{6}{5} > 1$.

20. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}} = 3\sum_{n=1}^{\infty} \left(\frac{e}{3}\right)^n$ is a geometric series with first term $3(e/3) = e$ and ratio $r = \dfrac{e}{3}$. Since $|r| < 1$, the series converges. Its sum is $\dfrac{e}{1 - e/3} = \dfrac{3e}{3-e}$.

24. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)^2}{n(n+2)}$ diverges by (7), the Test for Divergence, since

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^2+2n+1}{n^2+2n} = \lim_{n\to\infty} \left(1 + \frac{1}{n^2+2n}\right) = 1 \neq 0.$$

38. $6.2\overline{54} = 6.2 + \dfrac{54}{10^3} + \dfrac{54}{10^5} + \cdots = 6.2 + \dfrac{54/10^3}{1 - 1/10^2} = \dfrac{62}{10} + \dfrac{54}{990} = \dfrac{6192}{990} = \dfrac{344}{55}$

44. $\displaystyle\sum_{n=0}^{\infty} \frac{(x+3)^n}{2^n}$ is a geometric series with $r = \dfrac{x+3}{2}$, so the series converges $\iff |r| < 1 \iff \dfrac{|x+3|}{2} < 1 \iff$ $|x+3| < 2 \iff -5 < x < -1$. For these values of x, the sum of the series is $\dfrac{1}{1-(x+3)/2} = \dfrac{2}{2-(x+3)} = -\dfrac{2}{x+1}$.

49. For $n = 1$, $a_1 = 0$ since $s_1 = 0$. For $n > 1$,

$$a_n = s_n - s_{n-1} = \frac{n-1}{n+1} - \frac{(n-1)-1}{(n-1)+1} = \frac{(n-1)n - (n+1)(n-2)}{(n+1)n} = \frac{2}{n(n+1)}$$

Also, $\displaystyle\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{1 - 1/n}{1 + 1/n} = 1$.