8. $\displaystyle\int \frac{r^2}{r+4}\,dr = \int\left(\frac{r^2-16}{r+4}+\frac{16}{r+4}\right)dr = \int\left(r-4+\frac{16}{r+4}\right)dr$ [or use long division]

$= \frac{1}{2}r^2 - 4r + 16\ln|r+4| + C$

12. $\dfrac{x-1}{x^2+3x+2} = \dfrac{A}{x+1}+\dfrac{B}{x+2}$. Multiply both sides by $(x+1)(x+2)$ to get $x-1 = A(x+2)+B(x+1)$. Substituting -2 for x gives $-3=-B \Leftrightarrow B=3$. Substituting -1 for x gives $-2=A$. Thus,

$$\int_0^1 \frac{x-1}{x^2+3x+2}\,dx = \int_0^1\left(\frac{-2}{x+1}+\frac{3}{x+2}\right)dx = \left[-2\ln|x+1|+3\ln|x+2|\right]_0^1$$

$$= (-2\ln 2 + 3\ln 3) - (-2\ln 1 + 3\ln 2) = 3\ln 3 - 5\ln 2 \quad \left[\text{or } \ln\tfrac{27}{32}\right]$$

18. $\dfrac{x^2+2x-1}{x^3-x} = \dfrac{x^2+2x-1}{x(x+1)(x-1)} = \dfrac{A}{x}+\dfrac{B}{x+1}+\dfrac{C}{x-1}$. Multiply both sides by $x(x+1)(x-1)$ to get $x^2+2x-1 = A(x+1)(x-1)+Bx(x-1)+Cx(x+1)$. Substituting 0 for x gives $-1=-A \Leftrightarrow A=1$. Substituting -1 for x gives $-2=2B \Leftrightarrow B=-1$. Substituting 1 for x gives $2=2C \Leftrightarrow C=1$. Thus,

$$\int \frac{x^2+2x-1}{x^3-x}\,dx = \int\left(\frac{1}{x}-\frac{1}{x+1}+\frac{1}{x-1}\right)dx = \ln|x|-\ln|x+1|+\ln|x-1|+C = \ln\left|\frac{x(x-1)}{x+1}\right|+C.$$

26. $\dfrac{x^2-x+6}{x^3+3x} = \dfrac{x^2-x+6}{x(x^2+3)} = \dfrac{A}{x}+\dfrac{Bx+C}{x^2+3}$. Multiply by $x(x^2+3)$ to get $x^2-x+6 = A(x^2+3)+(Bx+C)x$. Substituting 0 for x gives $6=3A \Leftrightarrow A=2$. The coefficients of the x^2-terms must be equal, so $1=A+B \Rightarrow B=1-2=-1$. The coefficients of the x-terms must be equal, so $-1=C$. Thus,

$$\int \frac{x^2-x+6}{x^3+3x}\,dx = \int\left(\frac{2}{x}+\frac{-x-1}{x^2+3}\right)dx = \int\left(\frac{2}{x}-\frac{x}{x^2+3}-\frac{1}{x^2+3}\right)dx$$

$$= 2\ln|x| - \frac{1}{2}\ln(x^2+3) - \frac{1}{\sqrt{3}}\tan^{-1}\frac{x}{\sqrt{3}} + C$$

34. $\dfrac{x^3}{x^3+1} = \dfrac{(x^3+1)-1}{x^3+1} = 1-\dfrac{1}{x^3+1} = 1-\left(\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2-x+1}\right) \Rightarrow$ $1 = A(x^2-x+1)+(Bx+C)(x+1)$. Equate the terms of degree 2, 1 and 0 to get $0=A+B$, $0=-A+B+C$, $1=A+C$. Solve the three equations to get $A=\frac{1}{3}$, $B=-\frac{1}{3}$, and $C=\frac{2}{3}$. So

$$\int \frac{x^3}{x^3+1}\,dx = \int\left[1-\frac{\frac{1}{3}}{x+1}+\frac{\frac{1}{3}x-\frac{2}{3}}{x^2-x+1}\right]dx$$

$$= x - \tfrac{1}{3}\ln|x+1| + \frac{1}{6}\int\frac{2x-1}{x^2-x+1}\,dx - \frac{1}{2}\int\frac{dx}{\left(x-\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$= x - \tfrac{1}{3}\ln|x+1| + \tfrac{1}{6}\ln(x^2-x+1) - \tfrac{1}{\sqrt{3}}\tan^{-1}\left(\tfrac{1}{\sqrt{3}}(2x-1)\right) + K$$

3. $\int_0^2 \frac{2t}{(t-3)^2}\,dt = \int_{-3}^{-1} \frac{2(u+3)}{u^2}\,du \;\; [u = t-3,\, du = dt] \;\; = \int_{-3}^{-1}\left(\frac{2}{u} + \frac{6}{u^2}\right) du = \left[2\ln|u| - \frac{6}{u}\right]_{-3}^{-1}$

$= (2\ln 1 + 6) - (2\ln 3 + 2) = 4 - 2\ln 3$ or $4 - \ln 9$

5. Let $u = \arctan y$. Then $du = \dfrac{dy}{1+y^2} \;\Rightarrow\; \int_{-1}^{1} \frac{e^{\arctan y}}{1+y^2}\,dy = \int_{-\pi/4}^{\pi/4} e^u\,du = \left[e^u\right]_{-\pi/4}^{\pi/4} = e^{\pi/4} - e^{-\pi/4}$.

16. $\int_0^{\sqrt{2}/2} \frac{x^2}{\sqrt{1-x^2}}\,dx = \int_0^{\pi/4} \frac{\sin^2\theta}{\cos\theta}\cos\theta\,d\theta \qquad [x = \sin\theta,\, dx = \cos\theta\,d\theta]$

$= \int_0^{\pi/4} \frac{1}{2}(1 - \cos 2\theta)\,d\theta = \frac{1}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\pi/4} = \frac{1}{2}\left[\left(\frac{\pi}{4} - \frac{1}{2}\right) - (0 - 0)\right] = \frac{\pi}{8} - \frac{1}{4}$

[illegible] Let $u = 1 - x^2$. Then $du = -2x\,dx \;\Rightarrow$

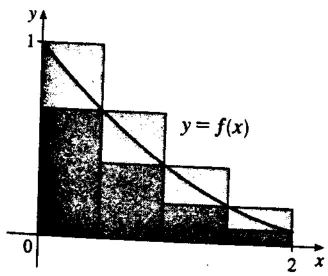
$$\int \frac{x\,dx}{1 - x^2 + \sqrt{1-x^2}} = -\frac{1}{2}\int \frac{du}{u + \sqrt{u}} = -\int \frac{v\,dv}{v^2 + v} \quad [v = \sqrt{u},\, u = v^2,\, du = 2v\,dv]$$

$$= -\int \frac{dv}{v+1} = -\ln|v+1| + C = -\ln\left(\sqrt{1-x^2} + 1\right) + C$$

41. Let $u = \theta$, $dv = \tan^2\theta\,d\theta = \left(\sec^2\theta - 1\right)d\theta \;\Rightarrow\; du = d\theta$ and $v = \tan\theta - \theta$. So

$$\int \theta\tan^2\theta\,d\theta = \theta(\tan\theta - \theta) - \int(\tan\theta - \theta)\,d\theta = \theta\tan\theta - \theta^2 - \ln|\sec\theta| + \tfrac{1}{2}\theta^2 + C$$

$$= \theta\tan\theta - \tfrac{1}{2}\theta^2 - \ln|\sec\theta| + C$$

2. 

The diagram shows that $L_4 > T_4 > \int_0^2 f(x)\,dx > R_4$, and it appears that M_4 is a bit less than $\int_0^2 f(x)\,dx$. In fact, for any function that is concave upward, it can be shown that

$L_n > T_n > \int_0^2 f(x)\,dx > M_n > R_n$.

(a) Since $0.9540 > 0.8675 > 0.8632 > 0.7811$, it follows that $L_n = 0.9540$, $T_n = 0.8675$, $M_n = 0.8632$, and $R_n = 0.7811$.

(b) Since $M_n < \int_0^2 f(x)\,dx < T_n$, we have $0.8632 < \int_0^2 f(x)\,dx < 0.8675$.

10. $f(t) = \dfrac{1}{1+t^2+t^4}$, $\Delta t = \dfrac{3-0}{6} = \dfrac{1}{2}$

(a) $T_6 = \frac{1}{2\cdot 2}\left[f(0) + 2f(\frac{1}{2}) + 2f(1) + 2f(\frac{3}{2}) + 2f(2) + 2f(\frac{5}{2}) + f(3)\right] \approx 0.895122$

(b) $M_6 = \frac{1}{2}\left[f(\frac{1}{4}) + f(\frac{3}{4}) + f(\frac{5}{4}) + f(\frac{7}{4}) + f(\frac{9}{4}) + f(\frac{11}{4})\right] \approx 0.895478$

(c) $S_6 = \frac{1}{2\cdot 3}\left[f(0) + 4f(\frac{1}{2}) + 2f(1) + 4f(\frac{3}{2}) + 2f(2) + 4f(\frac{5}{2}) + f(3)\right] \approx 0.898014$

20. (a) $T_8 = \frac{1}{8\cdot 2}\{f(0) + 2[f(\frac{1}{8}) + f(\frac{2}{8}) + \cdots + f(\frac{7}{8})] + f(1)\} \approx 0.902333$
$M_8 = \frac{1}{8}\left[f(\frac{1}{16}) + f(\frac{3}{16}) + f(\frac{5}{16}) + \cdots + f(\frac{15}{16})\right] = 0.905620$

(b) $f(x) = \cos(x^2)$, $f'(x) = -2x\sin(x^2)$, $f''(x) = -2\sin(x^2) - 4x^2\cos(x^2)$. For $0 \le x \le 1$, sin and cos are positive, so $|f''(x)| = 2\sin(x^2) + 4x^2\cos(x^2) \le 2\cdot 1 + 4\cdot 1\cdot 1 = 6$ since $\sin(x^2) \le 1$ and $\cos(x^2) \le 1$ for all x, and $x^2 \le 1$ for $0 \le x \le 1$. So for $n = 8$, we take $K = 6$, $a = 0$, and $b = 1$ in Theorem 3, to get $|E_T| \le 6\cdot 1^3/(12\cdot 8^2) = \frac{1}{128} = 0.0078125$ and $|E_M| \le \frac{1}{256} = 0.00390625$. [A better estimate is obtained by noting from a graph of f'' that $|f''(x)| \le 4$ for $0 \le x \le 1$.]

(c) Using $K = 6$ as in part (b), we have $|E_T| \le 6\cdot 1^3/(12n^2) = 1/(2n^2) \le 10^{-5} \Rightarrow 2n^2 \ge 10^5 \Rightarrow n \ge \sqrt{\frac{1}{2}\cdot 10^5}$ or $n \ge 224$. To guarantee that $|E_M| \le 0.00001$, we need $6\cdot 1^3/(24n^2) \le 10^{-5} \Rightarrow 4n^2 > 10^5 \Rightarrow n \ge \sqrt{\frac{1}{4}\cdot 10^5}$ or $n \ge 159$.

22. From Example 7(b), we take $K = 76e$ to get $|E_S| \le 76e(1)^5/(180n^4) \le 0.00001 \Rightarrow n^4 \ge 76e/[180(0.00001)] \Rightarrow n \ge 18.4$. Take $n = 20$ (since n must be even).

30. If x = distance from left end of pool and $w = w(x)$ = width at x, then Simpson's Rule with $n = 8$ and $\Delta x = 2$ gives Area $= \int_0^{16} w\,dx \approx \frac{2}{3}[0 + 4(6.2) + 2(7.2) + 4(6.8) + 2(5.6) + 4(5.0) + 2(4.8) + 4(4.8) + 0] \approx 84\ \text{m}^2$.