23. $\int_1^4 \sqrt{t}\,(1+t)\,dt = \int_1^4 (t^{1/2}+t^{3/2})\,dt = \left[\frac{2}{3}t^{3/2}+\frac{2}{5}t^{5/2}\right]_1^4 = \left(\frac{16}{3}+\frac{64}{5}\right)-\left(\frac{2}{3}+\frac{2}{5}\right) = \frac{14}{3}+\frac{62}{5} = \frac{256}{15}$

32. $\int_{\pi/4}^{\pi/3} \sec\theta\tan\theta\,d\theta = \left[\sec\theta\right]_{\pi/4}^{\pi/3} = \sec\frac{\pi}{3}-\sec\frac{\pi}{4} = 2-\sqrt{2}$

34. $$\int_0^{\pi/3} \frac{\sin\theta+\sin\theta\tan^2\theta}{\sec^2\theta}\,d\theta = \int_0^{\pi/3}\frac{\sin\theta\,(1+\tan^2\theta)}{\sec^2\theta}\,d\theta = \int_0^{\pi/3}\frac{\sin\theta\sec^2\theta}{\sec^2\theta}\,d\theta = \int_0^{\pi/3}\sin\theta\,d\theta$$

39. $\int_{-1}^{2}(x-2|x|)\,dx = \int_{-1}^{0}[x-2(-x)]\,dx + \int_0^2[x-2(x)]\,dx = \int_{-1}^{0}3x\,dx + \int_0^2(-x)\,dx = 3\left[\frac{1}{2}x^2\right]_{-1}^{0} - \left[\frac{1}{2}x^2\right]_0^2$
$= 3\left(0-\frac{1}{2}\right)-(2-0) = -\frac{7}{2} = -3.5$

52. Let $u = x^2$, so $du = 2x\,dx$. When $x=0$, $u=0$; when $x=\sqrt{\pi}$, $u=\pi$. Thus,
$\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx = \int_0^{\pi}\cos u\left(\frac{1}{2}\,du\right) = \frac{1}{2}[\sin u]_0^{\pi} = \frac{1}{2}(\sin\pi-\sin 0) = \frac{1}{2}(0-0) = 0.$

57. Let $u = 1/x$, so $du = -1/x^2\,dx$. When $x=1$, $u=1$; when $x=2$, $u=\frac{1}{2}$. Thus,
$$\int_1^2 \frac{e^{1/x}}{x^2}\,dx = \int_1^{1/2} e^u\,(-du) = -\left[e^u\right]_1^{1/2} = -(e^{1/2}-e) = e-\sqrt{e}.$$

65. Let $u=\ln x$, so $du = \dfrac{dx}{x}$. When $x=e$, $u=1$; when $x=e^4$; $u=4$. Thus,
$$\int_e^{e^4}\frac{dx}{x\sqrt{\ln x}} = \int_1^4 u^{-1/2}\,du = 2\left[u^{1/2}\right]_1^4 = 2(2-1) = 2.$$

70. $\int_{-a}^{a} x\sqrt{x^2+a^2}\,dx = 0$ by Theorem 7(b), since $f(x) = x\sqrt{x^2+a^2}$ is an odd function.

6. Let $u=t$, $dv = \sin 2t\,dt \;\Rightarrow\; du = dt$, $v = -\frac{1}{2}\cos 2t$. Then
$\int t\sin 2t\,dt = -\frac{1}{2}t\cos 2t + \frac{1}{2}\int\cos 2t\,dt = -\frac{1}{2}t\cos 2t + \frac{1}{4}\sin 2t + C.$

12. Let $u = \ln p$, $dv = p^5\,dp \;\Rightarrow\; du = \dfrac{1}{p}\,dp$, $v = \frac{1}{6}p^6$. Then
$\int p^5\ln p\,dp = \frac{1}{6}p^6\ln p - \frac{1}{6}\int p^5\,dp = \frac{1}{6}p^6\ln p - \frac{1}{36}p^6 + C.$

21. Let $u=\ln x$, $dv = x^{-2}\,dx \;\Rightarrow\; du = \dfrac{1}{x}\,dx$, $v = -x^{-1}$. By (6),
$$\int_1^2\frac{\ln x}{x^2}\,dx = \left[-\frac{\ln x}{x}\right]_1^2 + \int_1^2 x^{-2}\,dx = -\frac{1}{2}\ln 2 + \ln 1 + \left[-\frac{1}{x}\right]_1^2 = -\frac{1}{2}\ln 2 + 0 - \frac{1}{2} + 1 = \frac{1}{2} - \frac{1}{2}\ln 2.$$

30. Let $u = r^2$, $dv = \dfrac{r}{\sqrt{4+r^2}}\,dr \;\Rightarrow\; du = 2r\,dr$, $v = \sqrt{4+r^2}$. By (6),
$$\int_0^1\frac{r^3}{\sqrt{4+r^2}}\,dr = \left[r^2\sqrt{4+r^2}\right]_0^1 - 2\int_0^1 r\sqrt{4+r^2}\,dr = \sqrt{5} - \frac{2}{3}\left[(4+r^2)^{3/2}\right]_0^1$$
$$= \sqrt{5} - \frac{2}{3}(5)^{3/2} + \frac{2}{3}(8) = \sqrt{5}\left(1-\frac{10}{3}\right) + \frac{16}{3} = \frac{16}{3} - \frac{7}{3}\sqrt{5}$$

34. Let $w = \sqrt{x}$, so that $x = w^2$ and $dx = 2w\,dw$. Thus, $\int_1^4 e^{\sqrt{x}}\,dx = \int_1^2 e^w 2w\,dw$. Now use parts with $u = 2w$, $dv = e^w\,dw$, $du = 2\,dw$, $v = e^w$ to get $\int_1^2 e^w 2w\,dw = [2we^w]_1^2 - 2\int_1^2 e^w\,dw = 4e^2 - 2e - 2(e^2-e) = 2e^2.$