22. $\int_0^4 (2v+5)(3v-1)\,dv = \int_0^4 (6v^2+13v-5)\,dv = \left[6\cdot\frac{1}{3}v^3 + 13\cdot\frac{1}{2}v^2 - 5v\right]_0^4 = \left[2v^3 + \frac{13}{2}v^2 - 5v\right]_0^4$
$= (128 + 104 - 20) - 0 = 212$

28. $\int_0^5 (2e^x + 4\cos x)\,dx = [2e^x + 4\sin x]_0^5 = (2e^5 + 4\sin 5) - (2e^0 + 4\sin 0) = 2e^5 + 4\sin 5 - 2 \approx 290.99$

33. $\displaystyle\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta = \int_0^{\pi/4}\left(\frac{1}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}\right)d\theta = \int_0^{\pi/4}(\sec^2\theta + 1)\,d\theta$
$= [\tan\theta + \theta]_0^{\pi/4} = \left(\tan\frac{\pi}{4} + \frac{\pi}{4}\right) - (0+0) = 1 + \frac{\pi}{4}$

40. $\int_0^{3\pi/2} |\sin x|\,dx = \int_0^{\pi} \sin x\,dx + \int_\pi^{3\pi/2}(-\sin x)\,dx = [-\cos x]_0^\pi + [\cos x]_\pi^{3\pi/2}$
$= [1-(-1)] + [0-(-1)] = 2 + 1 = 3$

51. Let $u = 1 + 2x^3$, so $du = 6x^2\,dx$. When $x = 0$, $u = 1$; when $x = 1$, $u = 3$. Thus,

$\int_0^1 x^2(1+2x^3)^5\,dx = \int_1^3 u^5\left(\frac{1}{6}\,du\right) = \frac{1}{6}\left[\frac{1}{6}u^6\right]_1^3 = \frac{1}{36}(3^6 - 1^6) = \frac{1}{36}(729-1) = \frac{728}{36} = \frac{182}{9}$

56. $\displaystyle\int_0^2 \frac{dx}{(2x-3)^2}$ does not exist since $f(x) = \dfrac{1}{(2x-3)^2}$ has an infinite discontinuity at $x = \frac{3}{2}$.

58. Let $u = -x^2$, so $du = -2x\,dx$. When $x = 0$, $u = 0$; when $x = 1$, $u = -1$. Thus,

$\int_0^1 xe^{-x^2}\,dx = \int_0^{-1} e^u\left(-\frac{1}{2}\,du\right) = -\frac{1}{2}\left[e^u\right]_0^{-1} = -\frac{1}{2}\left(e^{-1} - e^0\right) = \frac{1}{2}(1 - 1/e).$

62. Let $u = \sin x$, so $du = \cos x\,dx$. When $x = 0$, $u = 0$; when $x = \frac{\pi}{2}$, $u = 1$. Thus,

$\int_0^{\pi/2} \cos x\,\sin(\sin x)\,dx = \int_0^1 \sin u\,du = \left[-\cos u\right]_0^1 = -(\cos 1 - 1) = 1 - \cos 1.$

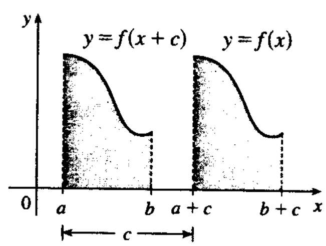
79. Let $u = 2x$. Then $du = 2\,dx$, so $\int_0^2 f(2x)\,dx = \int_0^4 f(u)\left(\frac{1}{2}\,du\right) = \frac{1}{2}\int_0^4 f(u)\,du = \frac{1}{2}(10) = 5.$

80. Let $u = x^2$. Then $du = 2x\,dx$, so $\int_0^3 xf(x^2)\,dx = \int_0^9 f(u)\left(\frac{1}{2}\,du\right) = \frac{1}{2}\int_0^9 f(u)\,du = \frac{1}{2}(4) = 2.$

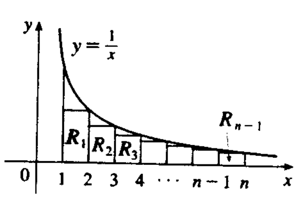
82. Let $u = x + c$. Then $du = dx$, so

$\int_a^b f(x+c)\,dx = \int_{a+c}^{b+c} f(u)\,du = \int_{a+c}^{b+c} f(x)\,dx$

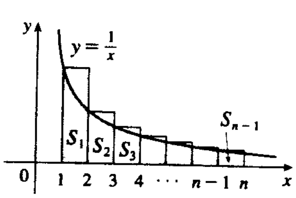
From the diagram, we see that the equality follows from the fact that we are translating the graph of f, and the limits of integration, by a distance c.



3.



The area of R_i is $\dfrac{1}{i+1}$ and so $\dfrac{1}{2} + \dfrac{1}{3} + \cdots + \dfrac{1}{n} < \displaystyle\int_1^n \frac{1}{t}\,dt = \ln n.$



The area of S_i is $\dfrac{1}{i}$ and so $1 + \dfrac{1}{2} + \cdots + \dfrac{1}{n-1} > \displaystyle\int_1^n \frac{1}{t}\,dt = \ln n.$