6. $y' = x + 5y \Rightarrow y' - 5y = x$. $I(x) = e^{\int P(x)dx} = e^{\int(-5)dx} = e^{-5x}$. Multiplying the differential equation by $I(x)$ gives $e^{-5x}y' - 5e^{-5x}y = xe^{-5x} \Rightarrow (e^{-5x}y)' = xe^{-5x} \Rightarrow$
$e^{-5x}y = \int xe^{-5x}\,dx = -\frac{1}{5}xe^{-5x} - \frac{1}{25}e^{-5x} + C$ [by parts] $\Rightarrow y = -\frac{1}{5}x - \frac{1}{25} + Ce^{5x}$

8. $x^2y' + 2xy = \cos^2 x \Rightarrow y' + \dfrac{2}{x}y = \dfrac{\cos^2 x}{x^2}$. $I(x) = e^{\int P(x)\,dx} = e^{\int 2/x\,dx} = e^{2\ln|x|} = e^{\ln(x^2)} = x^2$.
Multiplying by $I(x)$ gives us our original equation back. You may have noticed this immediately, since $P(x)$ is the derivative of the coefficient of y'. We rewrite it as $(x^2y)' = \cos^2 x$. Thus,
$x^2y = \int \cos^2 x\,dx = \int \frac{1}{2}(1 + \cos 2x)\,dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C \Rightarrow y = \dfrac{1}{2x} + \dfrac{1}{4x^2}\sin 2x + \dfrac{C}{x^2}$ or
$y = \dfrac{1}{2x} + \dfrac{1}{2x^2}\sin x\cos x + \dfrac{C}{x^2}$.

14. $2xy' + y = 6x,\ x > 0 \Rightarrow y' + \dfrac{1}{2x}y = 3$. $I(x) = e^{\int 1/(2x)\,dx} = e^{(1/2)\ln x} = e^{\ln x^{1/2}} = \sqrt{x}$. Multiplying by $\sqrt{x}$ gives $\sqrt{x}\,y' + \dfrac{1}{2\sqrt{x}}y = 3\sqrt{x} \Rightarrow (\sqrt{x}\,y)' = 3\sqrt{x} \Rightarrow \sqrt{x}\,y = \int 3\sqrt{x}\,dx = 2x^{3/2} + C \Rightarrow$
$y = 2x + \dfrac{C}{\sqrt{x}}$. $y(4) = 20 \Rightarrow 8 + \dfrac{C}{2} = 20 \Rightarrow C = 24$, so $y = 2x + \dfrac{24}{\sqrt{x}}$.

34. Let $y(t)$ denote the amount of chlorine in the tank at time t (in seconds). $y(0) = (0.05\text{ g/L})(400\text{ L}) = 20\text{ g}$. The amount of liquid in the tank at time t is $(400 - 6t)$ L since 4 L of water enters the tank each second and 10 L of liquid leaves the tank each second. Thus, the concentration of chlorine at time t is $\dfrac{y(t)}{400 - 6t}\,\dfrac{\text{g}}{\text{L}}$. Chlorine doesn't enter the tank, but it leaves at a rate of $\left[\dfrac{y(t)}{400 - 6t}\,\dfrac{\text{g}}{\text{L}}\right]\left[10\,\dfrac{\text{L}}{\text{s}}\right] = \dfrac{10\,y(t)}{400 - 6t}\,\dfrac{\text{g}}{\text{s}} = \dfrac{5\,y(t)}{200 - 3t}\,\dfrac{\text{g}}{\text{s}}$. Therefore,
$\dfrac{dy}{dt} = -\dfrac{5y}{200 - 3t} \Rightarrow \displaystyle\int \frac{dy}{y} = \int \frac{-5\,dt}{200 - 3t} \Rightarrow \ln y = \frac{5}{3}\ln(200 - 3t) + C \Rightarrow$
$y = \exp\left(\frac{5}{3}\ln(200 - 3t) + C\right) = e^C(200 - 3t)^{5/3}$. Now $20 = y(0) = e^C \cdot 200^{5/3} \Rightarrow e^C = \dfrac{20}{200^{5/3}}$, so
$y(t) = 20\dfrac{(200 - 3t)^{5/3}}{200^{5/3}} = 20(1 - 0.015t)^{5/3}$ g for $0 \le t \le 66\frac{2}{3}$ s, at which time the tank is empty.

35. (a) $\dfrac{dv}{dt} + \dfrac{c}{m}v = g$ and $I(t) = e^{\int(c/m)dt} = e^{(c/m)t}$, and multiplying the differential equation by $I(t)$ gives
$e^{(c/m)t}\dfrac{dv}{dt} + \dfrac{vce^{(c/m)t}}{m} = ge^{(c/m)t} \Rightarrow \left[e^{(c/m)t}v\right]' = ge^{(c/m)t}$. Hence,
$v(t) = e^{-(c/m)t}\left[\int ge^{(c/m)t}\,dt + K\right] = mg/c + Ke^{-(c/m)t}$. But the object is dropped from rest, so $v(0) = 0$ and $K = -mg/c$. Thus, the velocity at time t is $v(t) = (mg/c)\left[1 - e^{-(c/m)t}\right]$.

(b) $\lim\limits_{t\to\infty} v(t) = mg/c$

(c) $s(t) = \int v(t)\,dt = (mg/c)\left[t + (m/c)e^{-(c/m)t}\right] + c_1$ where $c_1 = s(0) - m^2g/c^2$. $s(0)$ is the initial position, so $s(0) = 0$ and $s(t) = (mg/c)\left[t + (m/c)e^{-(c/m)t}\right] - m^2g/c^2$.

36. $v = (mg/c)(1 - e^{-ct/m}) \Rightarrow \dfrac{dv}{dm} = \dfrac{mg}{c}\left(0 - e^{-ct/m} \cdot \dfrac{ct}{m^2}\right) + \dfrac{g}{c}\left(1 - e^{-ct/m}\right) \cdot 1 =$
$-\dfrac{gt}{m}e^{-ct/m} + \dfrac{g}{c} - \dfrac{g}{c}e^{-ct/m} = \dfrac{g}{c}\left(1 - e^{-ct/m} - \dfrac{ct}{m}e^{-ct/m}\right) \Rightarrow$
$\dfrac{c}{g}\dfrac{dv}{dm} = 1 - \left(1 + \dfrac{ct}{m}\right)e^{-ct/m} = 1 - \dfrac{1 + ct/m}{e^{ct/m}} = 1 - \dfrac{1 + Q}{e^Q}$, where $Q = \dfrac{ct}{m} \ge 0$. Since $e^Q > 1 + Q$ for all $Q > 0$, it follows that $dv/dm > 0$ for $t > 0$. In other words, for all $t > 0$, v increases as m increases.

3. $y' = y - 1$. The slopes at each point are independent of x, so the slopes are the same along each line parallel to the x-axis. Thus, IV is the direction field for this equation. Note that for $y = 1$, $y' = 0$.

4. $y' = y - x = 0$ on the line $y = x$, when $x = 0$ the slope is y, and when $y = 0$ the slope is $-x$. Direction field II satisfies these conditions. [Looking at the slope at the point $(0, 2)$, II looks more like it has a slope of 2 than does direction field I.]

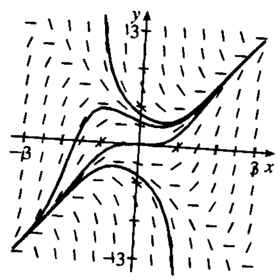
5. $y' = y^2 - x^2 = 0 \;\Rightarrow\; y = \pm x$. There are horizontal tangents on these lines only in graph III, so this equation corresponds to direction field III.

6. $y' = y^3 - x^3 = 0$ on the line $y = x$, when $x = 0$ the slope is y^3, and when $y = 0$ the slope is $-x^3$. The graph is similar to the graph for Exercise 4, but the segments must get steeper very rapidly as they move away from the origin, because x and y are raised to the third power. This is the case in direction field I.

[illegible]

x	y	$y' = x^2 - y^2$
± 1	± 3	-8
± 3	± 1	8
± 1	± 0.5	0.75
± 0.5	± 1	-0.75

Note that $y' = 0$ for $y = \pm x$. If $|x| < |y|$, then $y' < 0$; that is, the slopes are negative for all points in quadrants I and II above both of the lines $y = x$ and $y = -x$, and all points in quadrants III and IV below both of the lines $y = -x$ and $y = x$. A similar statement holds for positive slopes.



22. $h = 0.2$, $x_0 = 0$, $y_0 = 0$, and $F(x, y) = 1 - xy$.

Note that $x_1 = x_0 + h = 0 + 0.2 = 0.2$, $x_2 = 0.4$, $x_3 = 0.6$, and $x_4 = 0.8$.

$y_1 = y_0 + hF(x_0, y_0) = 0 + 0.2F(0, 0) = 0.2[1 - (0)(0)] = 0.2.$

$y_2 = y_1 + hF(x_1, y_1) = 0.2 + 0.2F(0.2, 0.2) = 0.2 + 0.2[1 - (0.2)(0.2)] = 0.392.$

$y_3 = y_2 + hF(x_2, y_2) = 0.392 + 0.2F(0.4, 0.392) = 0.392 + 0.2[1 - (0.4)(0.392)] = 0.56064.$

$y_4 = y_3 + hF(x_3, y_3) = 0.56064 + 0.2[1 - (0.6)(0.56064)] = 0.6933632.$

$y_5 = y_4 + hF(x_4, y_4) = 0.6933632 + 0.2[1 - (0.8)(0.6933632)] = 0.782425088.$

Thus, $y(1) \approx 0.7824$.