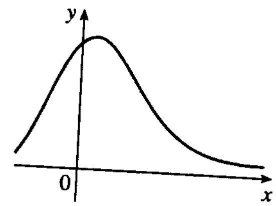
$y = e^{rt} \Rightarrow y' = re^{rt} \Rightarrow y'' = r^2e^{rt}$. $y'' + y' - 6y = 0 \Rightarrow r^2e^{rt} + re^{rt} - 6e^{rt} = 0 \Rightarrow$
$(r^2 + r - 6)e^{rt} = 0 \Rightarrow (r+3)(r-2) = 0 \Rightarrow r = -3$ or 2

10. (a) $y = k \Rightarrow y' = 0$, so $\dfrac{dy}{dt} = y^4 - 6y^3 + 5y^2 \Leftrightarrow 0 = k^4 - 6k^3 + 5k^2 \Leftrightarrow k^2(k^2 - 6k + 5) = 0 \Leftrightarrow$
$k^2(k-1)(k-5) = 0 \Leftrightarrow k = 0, 1,$ or 5

(b) y is increasing $\Leftrightarrow \dfrac{dy}{dt} > 0 \Leftrightarrow y^2(y-1)(y-5) > 0 \Leftrightarrow y \in (-\infty, 0) \cup (0, 1) \cup (5, \infty)$

(c) y is decreasing $\Leftrightarrow \dfrac{dy}{dt} < 0 \Leftrightarrow y \in (1, 5)$

12. The graph for this exercise is shown in the figure at the right.



A. $y' = 1 + xy > 1$ for points in the first quadrant, but we can see that $y' < 0$ for some points in the first quadrant. So equation A is incorrect.

B. $y' = -2xy = 0$ when $x = 0$, but we can see that $y' > 0$ for $x = 0$. So equation B is incorrect.

C. $y' = 1 - 2xy$ seems reasonable since:

(1) When $x = 0$, y' could be 1.

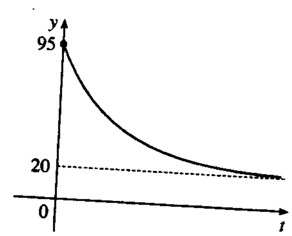
(2) When $x < 0$, y' could be greater than 1.

(3) Solving $y' = 1 - 2xy$ for y gives us $y = \dfrac{1 - y'}{2x}$. If y' takes on small negative values, then as $x \to \infty$, $y \to 0^+$, as shown in the figure. Thus, the correct equation is C.

14. (a) The coffee cools most quickly as soon as it is removed from the heat source. The rate of cooling decreases toward 0 since the coffee approaches room temperature.

(b) $\dfrac{dy}{dt} = k(y - R)$, where k is a proportionality constant, y is the temperature of the coffee, and R is the room temperature. The initial condition is $y(0) = 95\,°\text{C}$. The answer and the model support each other because as y approaches R, dy/dt approaches 0, so the model seems appropriate.

(c)



8. $y' = \dfrac{xy}{2\ln y} \Rightarrow \dfrac{2\ln y}{y}\,dy = x\,dx \Rightarrow \displaystyle\int \frac{2\ln y}{y}\,dy = \int x\,dx \Rightarrow (\ln y)^2 = \frac{x^2}{2} + C \Rightarrow$
$\ln y = \pm\sqrt{x^2/2 + C} \Rightarrow y = e^{\pm\sqrt{x^2/2 + C}}$

14. $\dfrac{dP}{dt} = \sqrt{Pt} \Rightarrow dP/\sqrt{P} = \sqrt{t}\,dt \Rightarrow \int P^{-1/2}\,dP = \int t^{1/2}\,dt \Rightarrow 2P^{1/2} = \frac{2}{3}t^{3/2} + C.$
$P(1) = 2 \Rightarrow 2\sqrt{2} = \frac{2}{3} + C \Rightarrow C = 2\sqrt{2} - \frac{2}{3}$, so $2P^{1/2} = \frac{2}{3}t^{3/2} + 2\sqrt{2} - \frac{2}{3} \Rightarrow$
$\sqrt{P} = \frac{1}{3}t^{3/2} + \sqrt{2} - \frac{1}{3} \Rightarrow P = \left(\frac{1}{3}t^{3/2} + \sqrt{2} - \frac{1}{3}\right)^2.$

18. $xy' + y = y^2 \;\Rightarrow\; x\dfrac{dy}{dx} = y^2 - y \;\Rightarrow\; x\,dy = (y^2 - y)\,dx \;\Rightarrow\; \dfrac{dy}{y^2 - y} = \dfrac{dx}{x} \;\Rightarrow$

$\displaystyle\int \frac{dy}{y(y-1)} = \int \frac{dx}{x}$ $[y \neq 0, 1]$ $\;\Rightarrow\; \displaystyle\int \left(\frac{1}{y-1} - \frac{1}{y}\right) dy = \int \frac{dx}{x} \;\Rightarrow\; \ln|y-1| - \ln|y| = \ln|x| + C$

$\Rightarrow\; \ln\left|\dfrac{y-1}{y}\right| = \ln\left(e^C|x|\right) \;\Rightarrow\; \left|\dfrac{y-1}{y}\right| = e^C|x| \;\Rightarrow\; \dfrac{y-1}{y} = Kx$, where $K = \pm e^C \;\Rightarrow$

$1 - \dfrac{1}{y} = Kx \;\Rightarrow\; \dfrac{1}{y} = 1 - Kx \;\Rightarrow\; y = \dfrac{1}{1 - Kx}$. [The excluded cases, $y = 0$ and $y = 1$, are ruled out by the initial condition $y(1) = -1$.] Now $y(1) = -1 \;\Rightarrow\; -1 = \dfrac{1}{1-K} \;\Rightarrow\; 1 - K = -1 \;\Rightarrow\; K = 2$,

so $y = \dfrac{1}{1 - 2x}$.

40. (a) If $y(t)$ is the amount of salt (in kg) after t minutes, then $y(0) = 0$ and the total amount of liquid in the tank remains constant at 1000 L.

$$\frac{dy}{dt} = \left(0.05\,\frac{\text{kg}}{\text{L}}\right)\left(5\,\frac{\text{L}}{\text{min}}\right) + \left(0.04\,\frac{\text{kg}}{\text{L}}\right)\left(10\,\frac{\text{L}}{\text{min}}\right) - \left(\frac{y(t)}{1000}\,\frac{\text{kg}}{\text{L}}\right)\left(15\,\frac{\text{L}}{\text{min}}\right)$$

$$= 0.25 + 0.40 - 0.015y = 0.65 - 0.015y = \frac{130 - 3y}{200}\,\frac{\text{kg}}{\text{min}}$$

so $\displaystyle\int \frac{dy}{130 - 3y} = \int \frac{dt}{200}$ and $-\frac{1}{3}\ln|130 - 3y| = \frac{1}{200}t + C$; since $y(0) = 0$, we have $-\frac{1}{3}\ln 130 = C$.

so $-\frac{1}{3}\ln|130 - 3y| = \frac{1}{200}t - \frac{1}{3}\ln 130 \;\Rightarrow\; \ln|130 - 3y| = -\frac{3}{200}t + \ln 130 = \ln\left(130e^{-3t/200}\right)$, and $|130 - 3y| = 130e^{-3t/200}$. Since y is continuous, $y(0) = 0$, and the right-hand side is never zero, we deduce that $130 - 3y$ is always positive. Thus, $130 - 3y = 130e^{-3t/200}$ and $y = \frac{130}{3}\left(1 - e^{-3t/200}\right)$ kg.

(b) After one hour, $y = \frac{130}{3}\left(1 - e^{-3\cdot 60/200}\right) = \frac{130}{3}\left(1 - e^{-0.9}\right) \approx 25.7$ kg.

Note: As $t \to \infty$, $y(t) \to \frac{130}{3} = 43\frac{1}{3}$ kg.

[illegible] (a) $y(t) = y(0)e^{kt} \;\Rightarrow\; y(2) = y(0)e^{2k} = 600$, $y(8) = y(0)e^{8k} = 75{,}000$. Dividing these equations, we get $e^{8k}/e^{2k} = 75{,}000/600 \;\Rightarrow\; e^{6k} = 125 \;\Rightarrow\; 6k = \ln 125 = \ln 5^3 = 3\ln 5 \;\Rightarrow\; k = \frac{3}{6}\ln 5 = \frac{1}{2}\ln 5$.
Thus, $y(0) = 600/e^{2k} = 600/e^{\ln 5} = \frac{600}{5} = 120$.

(b) $y(t) = y(0)e^{kt} = 120e^{(\ln 5)t/2}$ or $y = 120 \cdot 5^{t/2}$

(c) $y(5) = 120 \cdot 5^{5/2} = 120 \cdot 25\sqrt{5} = 3000\sqrt{5} \approx 6708$ bacteria.

(d) $y(t) = 120 \cdot 5^{t/2} \;\Rightarrow\; y'(t) = 120 \cdot 5^{t/2} \cdot \ln 5 \cdot \frac{1}{2} = 60 \cdot \ln 5 \cdot 5^{t/2}$.
$y'(5) = 60 \cdot \ln 5 \cdot 5^{5/2} = 60 \cdot \ln 5 \cdot 25\sqrt{5} \approx 5398$ bacteria/hour.

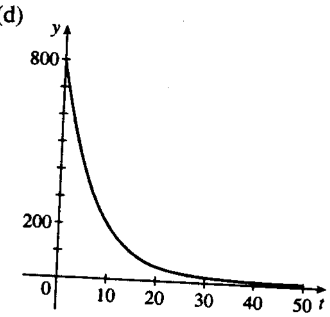
(e) $y(t) = 200{,}000 \;\Leftrightarrow\; 120e^{(\ln 5)t/2} = 200{,}000 \;\Leftrightarrow\; e^{(\ln 5)t/2} = \frac{5000}{3} \;\Leftrightarrow\; (\ln 5)t/2 = \ln\frac{5000}{3} \;\Leftrightarrow$
$t = \left(2\ln\frac{5000}{3}\right)/\ln 5 \approx 9.2$ h.

[illegible] (a) The mass remaining after t days is
$y(t) = y(0)e^{kt} = 800e^{kt}$. Since the half-life is 5.0 days,
$y(5) = 800e^{5k} = 400 \;\Rightarrow\; e^{5k} = \frac{1}{2} \;\Rightarrow$
$5k = \ln\frac{1}{2} \;\Rightarrow\; k = -(\ln 2)/5$, so
$y(t) = 800e^{-(\ln 2)t/5} = 800 \cdot 2^{-t/5}$.

(b) $y(30) = 800 \cdot 2^{-30/5} = 12.5$ mg

(c) $800e^{-(\ln 2)t/5} = 1 \;\Leftrightarrow\; -(\ln 2)\frac{t}{5} = \ln\frac{1}{800} = -\ln 800$
$\Leftrightarrow\; t = 5\,\frac{\ln 800}{\ln 2} \approx 48$ days

(d)



12. From the information given, we know that $\frac{dy}{dx} = 2y \quad \Rightarrow \quad y = Ce^{2x}$ by Theorem 2. To calculate C we use the point $(0, 5)$: $5 = Ce^{2(0)} \quad \Rightarrow \quad C = 5$. Thus, the equation of the curve is $y = 5e^{2x}$.

16. $\frac{dT}{dt} = k(T - 20)$. Let $y = T - 20$. Then $\frac{dy}{dt} = ky$, so $y(t) = y(0)e^{kt}$. $y(0) = T(0) - 20 = 95 - 20 = 75$, so $y(t) = 75e^{kt}$. When $T(t) = 70$, $\frac{dT}{dt} = -1°\text{C/min}$. Equivalently, $\frac{dy}{dt} = -1$ when $y(t) = 50$. Thus, $-1 = \frac{dy}{dt} = ky(t) = 50k$ and $50 = y(t) = 75e^{kt}$. The first relation implies $k = -1/50$, so the second relation says $50 = 75e^{-t/50}$. Thus, $e^{-t/50} = \frac{2}{3} \quad \Rightarrow \quad -t/50 = \ln\left(\frac{2}{3}\right) \quad \Rightarrow \quad t = -50\ln\left(\frac{2}{3}\right) \approx 20.27$ min.