Calculus I, 21-111
Review problems for the third test
April 25

1. Solve each of the following equations for x:
   (a) $\left(e^{3x} \cdot e^{-x}\right)^2 = e^6$
   (b) $\ln(3x+6) = 4$
   (c) $\ln(2)e^{-.3x} = 1$
2. Differentiate each of the following:
   (a) $y = \left(x^2 \ln(x)\right)^2$.
   (b) $y = (2x^3 + 4x)e^{4x}$.
   (c) $y = \ln(1 + e^x)$
   (d) $y = e^{-x^2/2}$
3. Sketch a graph of the function $f(x) = e^x - x$, labeling all maximums, minimums, and points of inflection.
4. Find the equation of the tangent line to the graph of $y = 3xe^x$ at $x = 0$.
5. From January 1, 1990 to January 1, 1997, the population of Texas grew from 17 million to 19.3 million.
   (a) Assuming exponential growth, give a formula for population t years after 1990.
   (b) According to this model, how large would the population be on Jan.1, 2005? (It was estimated to be 22,859,968 by the census bureau.)
   (c) According to the model, in what year will the population reach 30 million?
   (d) What will the growth rate of the population be at that time?
6. The half-life of cobalt-60 is 5.3 years. Suppose there is 15 grams of it.
   (a) How much will still be radioactive in 10 years?
   (b) How long will it take until less than 1 gram is radioavtive?
7. Suppose you invest \$5000 in a savings account.
   (a) If the account gives 6% annual interest, compounded monthly, how much will the account be worth in 5 years?
   (b) Suppose the 6% annual interest is compounded continuously. How much would the account be worth in 5 years?
   (c) In the continuous growth scenario, how fast is your money growing at time 0? At time 5 years?

8. Suppose you borrow \$100,000 to buy a house. The bank charges you 7% yearly interest, compounded monthly. If you plan to pay off the loan in 20 years, what will your monthly payment be?

9. Suppose you put \$4000 into a retirement savings account every year. The account pays 5% yearly interest. How much money will you have after 25 years?

10. Suppose someone offers to pay you \$15,000 in 10 years. Assuming you can safely invest money with a 4% yearly rate of return, what is the present value of their offer?

11. The decibel scale measures the loudness of sound according to the formula

$$\text{loundess} = 10 \log_{10} \left(\frac{x}{I_0} \right)$$

where x is the intensity of the sound measured in watts per square meter, and $I_0 = 10^{-12}$ watts per square meter is the least intense sound that a human ear can detect. How many times louder is a 108-decibel sound than a 100-decibel sound?