

Calculus I 21-111  
Skills Assessment Solutions  
Due January 18

Name:

**Note:** This assessment will not in any way affect your grade.

1. Simplify each of the following expressions as much as possible:

(a)  $x(y + z) - z(x + y) + 2y(x - z) - 3(3y - 2z)$

$$\begin{aligned}x(y + z) - z(x + y) + 2y(x - z) - 3(3y - 2z) &= xy + xz - zx - zy + 2yx - 2yz - 9y + 6z \\&= 3xy - 3yz - 9y + 6z\end{aligned}$$

(b)  $\frac{2}{5} - \frac{1}{2} + \frac{1}{3}$

$$\begin{aligned}\frac{2}{5} - \frac{1}{2} + \frac{1}{3} &= \frac{12}{30} - \frac{15}{30} + \frac{10}{30} \\&= \frac{12 - 15 + 10}{30} \\&= \frac{7}{30}\end{aligned}$$

(c)  $\frac{x}{x+2} - \frac{2}{x+1}$

$$\begin{aligned}\frac{x}{x+2} - \frac{2}{x+1} &= \frac{x(x+1)}{(x+2)(x+1)} - \frac{2(x+2)}{(x+1)(x+2)} \\&= \frac{x^2 + x}{(x+1)(x+2)} - \frac{2x + 4}{(x+1)(x+2)} \\&= \frac{x^2 + x - 2x - 4}{(x+1)(x+2)} \\&= \frac{x^2 - x - 4}{x^2 + 3x + 2}\end{aligned}$$

$$(d) \quad 2^{-2}x^2y^{-2}(2z)^3x^{-2}(2y)^3z^5$$

$$\begin{aligned} 2^{-2}x^2y^{-2}(2z)^3x^{-2}(2y)^3z^5 &= 2^{-2}2^32^3x^2x^{-2}y^{-2}y^3z^3z^5 \\ &= 2^4yz^8 \end{aligned}$$

$$(e) \quad \frac{\left(\frac{1}{2}x^{-2}y^2\right)^2}{\left(\frac{xy}{y^{-1}}\right)^{-1}}$$

$$\begin{aligned} \frac{\left(\frac{1}{2}x^{-2}y^2\right)^2}{\left(\frac{xy}{y^{-1}}\right)^{-1}} &= \frac{\frac{1}{4}x^{-4}y^4}{\frac{y^{-1}}{xy}} \\ &= \frac{y^4xy}{4x^4y^{-1}} \\ &= \frac{y^6}{4x^3} \end{aligned}$$

$$(f) \quad \sqrt{a^2b^2} \text{ (assume } a > 0 \text{ and } b > 0)$$

$$\begin{aligned} \sqrt{a^2b^2} &= \sqrt{(ab)^2} \\ &= ab \end{aligned}$$

$$(g) \quad \sqrt{a^2 + b^2} \text{ (assume } a > 0 \text{ and } b > 0)$$

Cannot be simplified further.

$$(h) \quad \left(\frac{9a^8}{16b^4}\right)^{-\frac{1}{2}}$$

$$\begin{aligned} \left(\frac{9a^8}{16b^4}\right)^{-\frac{1}{2}} &= \left(\frac{16b^4}{9a^8}\right)^{\frac{1}{2}} \\ &= \frac{4b^2}{3a^4} \end{aligned}$$

2. Determine all solutions to the following equations:

(a)  $3x - 7 = 5$

$$\begin{aligned} 3x - 7 = 5 &\Rightarrow 3x = 12 \\ &\Rightarrow x = 4 \end{aligned}$$

(b)  $x^2 - 5x + 6 = 0$   
Factor:

$$\begin{aligned} x^2 - 5x + 6 = 0 &\Rightarrow (x - 3)(x - 2) = 0 \\ &\Rightarrow x = 2 \text{ or } 3 \end{aligned}$$

(c)  $2x^2 + 3x - 2 = 0$   
Use the quadratic formula:

$$\begin{aligned} x &= \frac{-3 \pm \sqrt{3^2 - (4)(2)(-2)}}{2(2)} \\ &= \frac{-3 \pm \sqrt{9 + 16}}{4} \\ &= \frac{-3 \pm 5}{4} \\ &= \frac{2}{4} \text{ or } \frac{-8}{4} \\ &= \frac{1}{2} \text{ or } 2 \end{aligned}$$

(d)  $\frac{2x+1}{5} + \frac{3x+2}{2} = x$

$$\begin{aligned} \frac{2x+1}{5} + \frac{3x+2}{2} = x &\Rightarrow 2(2x+1) + 5(3x+2) = 10x \\ &\Rightarrow 4x + 2 + 15x + 10 = 10x \\ &\Rightarrow 29x = -12 \\ &\Rightarrow x = \frac{-12}{29} \end{aligned}$$

3. Let  $f(x) = 3x + 1$ , and  $g(x) = \frac{1}{x-1}$ .

(a) Evaluate  $f(g(2))$ .

$$g(2) = \frac{1}{2-1} = 1.$$

$$f(1) = 3(1) + 1 = 4.$$

$$\text{So, } f(g(2)) = f(1) = 4.$$

(b) Evaluate  $g(f(2))$ .  $f(2) = 3(2) + 1 = 7$ .

$$g(7) = \frac{1}{7-1} = \frac{1}{6}.$$

$$\text{So, } g(f(2)) = g(7) = \frac{1}{6}.$$

(c) Give a simplified expression for  $g(f(x))$ .

$$g(f(x)) = \frac{1}{(3x+1)-1} = \frac{1}{3x}$$

4. Determine the equation of the line through the point  $(2, 3)$  and having slope  $-2$ .

$$\text{Using point-slope form: } y - 3 = -2(x - 2)$$

5. You pay \$21.20 for an item including a 6% sales tax in the cost. How much was the item without the tax?

If we let  $x$  represent the price without tax, we get

$$x(1.06) = 21.20$$

So,  $x = 21.20/1.06 = 20$ . The item cost \$20.

6. Make a rough sketch of each of these functions:  $f(x) = 2, g(x) = x, h(x) = x^2, j(x) = x^3, k(x) = -x^4, m(x) = 1/x, p(x) = |x|, q(x) = 2^x$ .