Department of Mathematical Sciences Carnegie Mellon University

21-366 Random Graphs Test 2 $\,$

You can use my book and you can quote theorems from the book.

Problem	Points	Score
1	33	
2	33	
3	34	
Total	100	

Q1: (33pts) Let $p = \frac{K \log n}{n}$ and suppose that n = 3m where m is an integer. Prove that there exist vertex disjoint paths P_1, P_2, \ldots, P_m of length two that cover all the vertices of $G = G_{n,p}$. (Hint: reduce the problem to that of finding two perfect matchings.)

Q2: (33pts)

Prove that w.h.p. the random r-regular graph $G_{n,r}$, r = O(1), does not contain a copy of K_4 .

Q3: (34pts)

Give upper and lower bounds for the threshold for the existence of a spanning copy of the diagram below in $G_{n,p}$.



This is supposed to be a set of n/2 squares linked together in a cycle. n = 20 in the diagram.