1. Let \((\alpha, \beta) \in \mathbb{R}^2\) be given. Let \(x\) be the unique noncontinuable solution of

\[
\begin{align*}
\dot{x}_1(t) &= e^{-t}x_1(t) + e^{-2t}x_2(t) \\
\dot{x}_2(t) &= -e^{-2t}x_1(t) + e^{-t}x_2(t) \\
x_1(0) &= \alpha, \ x_2(0) = \beta
\end{align*}
\]

and let \(y\) be the unique noncontinuable solution of

\[
\begin{align*}
\dot{y}_1(t) &= e^{-t}y_1(t) + e^{2t}y_2(t) \\
\dot{y}_2(t) &= -e^{2t}y_1(t) + e^{-t}y_2(t) \\
y_1(0) &= \alpha, \ y_2(0) = \beta.
\end{align*}
\]

In Problem 2 of Assignment 3, you showed that \([0, \infty) \subset \text{Dom}(x)\) and you studied the behavior as \(t \to \infty\) of \(\|x(t)\|_2\).

(a) Show that \([0, \infty) \subset \text{Dom}(y)\).

(b) What can you say about the behavior of \(\|y(t)\|_2\) as \(t \to \infty\)?

(c) What, if anything, is the difference between the behavior of \(x(t)\) as \(t \to \infty\) and the behavior of \(y(t)\) as \(t \to \infty\)?

2. Show that the system

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) - x_1(t) + \sin t \\
\dot{x}_2(t) &= -x_1(t) + (1 - x_1(t)^2 - x_2(t)^2)x_2(t)
\end{align*}
\]

has a \(2\pi\)-periodic solution.
3. Show that the system

\[ \dot{x}_1(t) = 2 + \cos t + (1 + x_2(t)^2) \left( 1 - x_1(t)^2 \right) \]

\[ \dot{x}_2(t) = 3 - \sin t + (2 + x_1(t)^2) \left( 4 - x_2(t)^2 \right) \]

has a $2\pi$-period solution.

4. Draw the phase portrait for the autonomous system

\[ \dot{x}_1 = x_2 \]

\[ \dot{x}_2 = -x_1 + x_1^3. \]

What can you deduce about the behavior of solutions form the phase portrait?

5. If you did not turn in a solution to Problem 4 on Assignment 3 or if you wish to revise your solution to that problem, you may do so now.