No notes, tables, books, calculators can be used during the test. Please show all your work.
Q1: (20=10+10 pts)

- Find a power series representation for the function \( f(x) = \frac{1}{4 + x^2} \) and determine the radius of convergence.

- Find the Maclaurin series for the function \( f(x) = \sin x \).
Q2: (20pts) Find the maximum rate of change of $f(x,y) = x^2e^{-y}$ at the point (2,0) and the direction in which it occurs. Compute the directional derivative at the same point in the direction $\pi/6$. 
Q3: (20pts) Write up the system of equations (i.e., partial derivatives of the Lagrangian set to 0) for the following optimization problem. (You do NOT have to solve it.)

A person is planning to divide her savings among three mutual funds. Fund 1 has an expected return of 10%, Fund 2 has a 11% expected return and Fund 3 has a 15% expected return. Her goal is a return of at least 12% while minimizing her risk. The risk function for an investment in this combination fund is

\[ f(x_1, x_2, x_3) = 2x_1^2 + 4x_2^2 + x_1x_2 + 8x_3^2 + 2x_2x_3, \]

where \( x_i \) is the respective proportion of her savings in fund \( i \). Determine the optimal proportions \( x_1, x_2, x_3 \) invested in each fund.
Q4: (20pts) Find the *absolute* minimum of
\[ f(x, y, z) = x^2 + y^2 + z^2 \]
subject to the constraint \( x + y + z \geq 9 \).
Method: Use the Lagrangian function \( L \) and the Karush-Kuhn-Tucker theorem (verify the main hypotheses) to conclude that the critical point yields a minimum.
Q5: (20pts) Use the Gauss-Jordan method to determine the inverse of the matrix

\[ A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{pmatrix} \]

Determine the solutions \( x_1, x_2, x_3 \) of the following system of equations.

\[
\begin{align*}
x_1 + 2x_3 &= 0 \\
x_2 + 2x_3 &= 1 \\
x_1 + 2x_2 &= 0
\end{align*}
\]