CHASING A FAST ROBBER

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Joint work with Alan Frieze and Michael Krivelevich
Question

How many cops are required to catch a single robber on a given connected graph $G$, with perfect information?
**Setting (Nowakowski, Winkler)**

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**Rules**

1. Cops choose starting positions first.
2. Then Robber chooses starting position.
3. Each cop either moves by 1 edge, or stays put.
4. Robber either moves by 1 edge, or stays put.
5. Repeat steps 3 and 4.

Cops win when a cop occupies the same vertex as Robber.

Remark: It is possible for Robber to win, e.g., if $G$ is a cycle on 4 or more vertices and there is only 1 cop.
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**Theorem (Aigner, Fromme 1984)**

The cop number of Manhattan is 3.

**Proof idea.** Cop maintains invariant: for every vertex $v \in P$, he is closer to $v$ than Robber is. (Possible since $P$ is a shortest path.)
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If $P$ is a shortest path between some $a, b \in G$, then one cop is sufficient to keep Robber off $P$. 
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The cop number of *any planar graph* is at most 3.

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**Projective plane graph**

There are $C_4$-free bipartite graphs with all degrees $\Theta(\sqrt{n})$.

**Corollary**

The cop number of a general graph can be as large as $\Omega(\sqrt{n})$. 

Proof. Let $G$ be a projective plane graph. Suppose there are fewer than $\delta(G)$ cops. Robber stays put, unless a cop moves to an adjacent vertex. Since no $C_3$ or $C_4$, total number of robber's neighbors dominated/occupied by cops is $<\delta$, so robber can escape. □
General graphs: lower bound

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**Upper bounds**

- Frankl (1987): $\frac{n}{\log n} \cdot \log \log n$
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- If there is vertex of degree $\geq t$, put one cop there and eliminate it and its neighborhood from the graph.
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**Variations**

- On $G_{n,p}$: Bollobás-Kun-Leader, Łuczak-Prałat.
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- Faster cop.
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- On $G_{n,p}$: Bollobás-Kun-Leader, Łuczak-Prałat.
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Theorem (FGKNS, 2010)

If the robber can traverse up to 2 edges per move, then $\Omega(\sqrt{\log n})$ cops are required to catch the robber on the $n \times n$ grid.

Questions

Suppose the robber can travel $R \geq 2$ edges per move. Are there graphs which require more than $\sqrt{n}$ cops?

Can one prove an $o(n)$ upper bound?

Remarks.

Projective plane graphs were used for the old lower bound, but these are hard to analyze for fast robber strategies. Previous upper bound arguments used diameter lemma, which does not apply for fast robber.
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Theorem 1 (Frieze, Krivelevich, L.)

Let $R$ be the robber’s speed. There exist $n$-vertex graphs which:

- require $n^{1 - \frac{1}{R-2}}$ cops to catch the robber, if $3 \leq R \leq \infty$;
- require $\frac{n}{800^2}$ cops to catch the robber, if $R = \infty$. 

Theorem 2 (Frieze, Krivelevich, L.)

For any $R \geq 1$ and any connected graph $G$ on $n$ vertices, $\frac{n}{\alpha \sqrt{\log \alpha n}}$ cops are sufficient to catch any speed-$R$ robber, where $\alpha = 1 + \frac{1}{R}$. This smoothly extends the best upper bound to fast robbers.
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A graph $G$ is a **c-expander** if every set $S$ of at most half the vertices has $|N(S) \setminus S| \geq c|S|$.
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**Observation**

If $G$ needs many cops, then $G$ is an expander.

**Justification:**

- If set $S$ does not expand, station cops on $|N(S) \setminus S| < c|S|$. 
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- If set $S$ does not expand, station cops on $|N(S) \setminus S| < c|S|$.
- The robber can never pass this barrier, so the problem reduces to either $S$ or $G \setminus (N(S) \cup S)$.
- Cost in cops is only $c$-fraction of vertices removed.
To show $pn$ cops suffice:

- May assume all degrees $\leq \frac{1}{p}$.
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- May assume all degrees $\leq \frac{1}{p}$.
- May assume all sets expand by factor $\geq (1 + \frac{1}{p})$. 

Randomly place cops at every vertex with probability $p$. Choose $T$ so that $(1 + \frac{1}{p})^T = \frac{1}{p}$. Hall's Theorem: every vertex within distance $RT$ from robber has distinct cop within distance $T$ from it. Catch robber in $T$ rounds. Works as long as $(1 + \frac{1}{p})^{RT} \ll n$. $\square$
Benefits of expansion

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- Catch robber in $T$ rounds.
- Works as long as $(\frac{1}{p})^{RT} \ll n$. 

□
Lower bound

**Robber strategy on** \( G_{n,p} \)

- Let \( C \) be vertices occupied by cops.
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- Let $C$ be vertices occupied by cops.
- Let $C^+$ be $C$, together with neighboring vertices.
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- Let $C$ be vertices occupied by cops.
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- Robber stays outside $C^+$.

Proof.

For $np = n$, show speed-$1$ robber can elude $n^{1-c}$ cops.

In $G_{n,p}$, any $H = G - C^+$ has $np$-core of size $0$.

Say robber is in core, and cops move. Let $C'$ be new cop positions. Since $C' \subset C^+$, robber can still move within $H$. New core also has size $0$, so it overlaps old core. By properties of $G_{n,p}$, robber can reach new core fast. □
Robber strategy on $G_{n,p}$

- Let $C$ be vertices occupied by cops.
- Let $C^+$ be $C$, together with neighboring vertices.
- Robber stays in $\frac{np}{3}$-core of $G \setminus C^+$. 

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Remarks.

- Our lower bound robber strategy is (necessarily) more complex, so we use $G_{n,p}$ instead of the projective plane.
- Our upper bound matches the first-order constants of Lu-Peng and Scott-Sudakov, using expansion instead of diameter.
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Open problems.

- Are $\omega(\sqrt{n})$ cops required to catch a speed-2 robber? Our bound only exceeds $\sqrt{n}$ for $R \geq 5$.
- What if cops and robber move at the same speed $R \geq 2$?