1. First, convince yourself that every tree has chromatic number $\chi \leq 2$. (You don’t need to write that first part up.) Next, show that every graph which is the superposition of two possibly-overlapping trees on the same vertex set has chromatic number $\chi \leq 4$. Then show that every superposition of three possibly-overlapping trees on the same vertex set has chromatic number $\chi \leq 8$. Finally, determine the minimum integer $C$ such that every superposition of three possibly-overlapping trees on the same vertex set has chromatic number $\chi \leq C$.

2. For some infinite set of values of $t$, prove the following statement: every $n$-vertex graph with average degree $d$ contains at least $nd^t$ distinct walks of length $t$. These are sequences of $t + 1$ (possibly repeated) vertices $v_0, \ldots, v_t$, where each $v_i v_{i+1}$ is an edge. Note that you should prove this result not up to a constant factor: you should get a bound of really at least $nd^t$.

3. Is it true that for every integer $n$, there is an $n$-vertex graph with at least $10^{-6} n^2$ edges, each of which is contained in at least one triangle, but with no edge in more than $10^6$ triangles?