1 Problems

**Putnam 2007/B4.** Let \( n \) be a positive integer. Find the number of pairs \( P, Q \) of polynomials with real coefficients such that
\[
(P(X))^2 + (Q(X))^2 = X^{2n} + 1
\]
and \( \deg P > \deg Q \).

**Putnam 2007/B5.** Let \( k \) be a positive integer. Prove that there exist polynomials \( P_0(n), P_1(n), \ldots, P_{k-1}(n) \) (which may depend on \( k \)) such that for any integer \( n \),
\[
\left\lfloor \frac{n}{k} \right\rfloor^k = P_0(n) + P_1(n) \left\lfloor \frac{n}{k} \right\rfloor + \cdots + P_{k-1}(n) \left\lfloor \frac{n}{k} \right\rfloor^{k-1}.
\]
(\( \lfloor a \rfloor \) means the largest integer \( \leq a \).)

**Putnam 2007/B6.** For each positive integer \( n \), let \( f(n) \) be the number of ways to make \( n! \) cents using an unordered collection of coins, each worth \( k! \) cents for some \( k, 1 \leq k \leq n \). Prove that for some constant \( C \), independent of \( n \),
\[
n^{n^2/2-Cn}e^{-n^2/4} \leq f(n) \leq n^{n^2/2+Cn}e^{-n^2/4}.
\]