1 Problems

Putnam 1985/B4. Let $C$ be the unit circle $x^2 + y^2 = 1$. A point $P$ is chosen randomly on the circumference $C$ and another point $Q$ is chosen randomly from the interior of $C$ (these points are chosen independently and uniformly over their domains). Let $R$ be the rectangle with sides parallel to the $x$ and $y$-axes with diagonal $PQ$. What is the probability that no point of $R$ lies outside of $C$?

Putnam 1985/B5. Evaluate

$$\int_0^\infty t^{-1/2} e^{-1985(t+t^{-1})} dt.$$

You may assume that

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}.$$

Putnam 1985/B6. Let $G$ be a finite set of real $n \times n$ matrices $\{M_i\}$, $1 \leq i \leq r$, which form a group under matrix multiplication. Suppose that $\sum_{i=1}^{r} \text{tr}(M_i) = 0$, where $\text{tr}(A)$ denotes the trace of the matrix $A$. Prove that $\sum_{i=1}^{r} M_i$ is the $n \times n$ zero matrix.