1 Problems

**Putnam 2002/A1.** Let $k$ be a fixed positive integer. The $n$-th derivative of $\frac{1}{x^k - 1}$ has the form $\frac{P_n(x)}{(x^k - 1)^{n+1}}$ where $P_n(x)$ is a polynomial. Find $P_n(1)$.

**Putnam 2002/A2.** Given any five points on a sphere, show that some four of them must lie on a closed hemisphere.

**Putnam 2002/A3.** Let $n \geq 2$ be an integer and $T_n$ be the number of non-empty subsets $S$ of $\{1, 2, 3, \ldots, n\}$ with the property that the average of the elements of $S$ is an integer. Prove that $T_n - n$ is always even.