1 Problems

**Putnam 1999/A1.** Find polynomials $f(x), g(x),$ and $h(x),$ if they exist, such that for all $x$,

$$|f(x)| - |g(x)| + h(x) = \begin{cases} -1 & \text{if } x < -1 \\ 3x + 2 & \text{if } -1 \leq x \leq 0 \\ -2x + 2 & \text{if } x > 0. \end{cases}$$

**Putnam 1999/A2.** Let $p(x)$ be a polynomial that is nonnegative for all real $x$. Prove that for some $k$, there are polynomials $f_1(x), \ldots, f_k(x)$ such that

$$p(x) = \sum_{j=1}^{k} (f_j(x))^2.$$ 

**Putnam 1999/A3.** Consider the power series expansion

$$\frac{1}{1 - 2x - x^2} = \sum_{n=0}^{\infty} a_n x^n.$$ 

Prove that, for each integer $n \geq 0$, there is an integer $m$ such that

$$a_n^2 + a_{n+1}^2 = a_m.$$